## Certified Federal Surveyors Certification Program



## Course 4 <br> Restoration of Lost Corners

## Course 4: Restoration of Lost Corners Study Guide

## COURSE DESCRIPTION:

## COURSE

 OBJECTIVES:
## COURSE INSTRUCTOR(S):

## VIDEO LECTURE

 TITLE:This course consists of four videos, some reading, and three exercises, on the "Restoration of Lost Corners". The legal, mathematical, and practical applications of the methods of proportioning, as found in the Manual of Surveying Instructions, are presented. Students will be able to address what corners control in most situations, how to proportion properly, what legal principles are involved when proportioning, and how to deal with the latitudinal curve. A lengthy discussion of convergence and curvature in the PLSS is also included.

Upon completion of this course, students will be able to:

- Define the three corner conditions listed in the Manual of Surveying Instructions
- Describe, identify applicability, and compute proportions using all methods
- Demonstrate an understanding of curvature in the PLSS

Dennis Mouland, Bureau of Land Management
Ron Scherler, Bureau of Land Management

Restoration of Lost Corners - Part 1 (40 minutes)

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|  | EXERCISE | $\square$ <br> DIAGRAM |  |  |  |  |  |

## Restoration of Lost Corners - Part 1



2009
BLM MANUAL Before you begin the course, read Chapter 2, Section 2-9 to 2-43 and Chapter 7 of the 2009 BLM Manual

web course New to Restoration of Lost Corners? There is an optional web course available, which is accessible from the course description page.

## Introduction

Well hello everyone this is Dennis Mouland once again and the infamous Mr. Scherler. Well, I don't know if I am infamous. We are here to give you course number four of the CFedS Program and it is on Restoration of Lost Corners.

A very important subject and one that we've even had difficulty explaining it and going through it here and this is actually our $3^{\text {rd }}$ iteration of this course because for various technical reasons and other things; but it is a complex subject in spite of what some folks may think they already know about it we personally know and many of you will discover if you don't already know this, that an awful lot of the proper methods as described in the 2009 Manual for the proportioning of lost corners are not well known and there is not really any software that does most of it correctly. So that is why we are going to spend some time on this and we are glad to

RESTORATIONOF LOST CORNERS

Certified Federal Surveyors Course No. 4

Instructors:
Ron Scherler
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## Restoration of Lost Corners - Part 1

## Objectives

I think the first thing that we will do is take a look at our objectives for this course. Given a lost corner situation, whatever type of corner it is, you will be able to: (1) select the correct method for restoring lost corners based on the 2009 Manual; (2) properly compute any lost corner position that is the math itself based on the 2009 Manual, and (3) you will be able to identify some special cases where proportioning rules may need to be modified.

We will tackle those things in the order that we move along, but leaving special cases at the end $\ldots$ one thing that I am a big believer in you cannot know or understand when you have an exception to the rules unless you know the rules like the back of your hand.

## Is it Really Lost?

So we are going to go through the rules first and then talk about those pretty rare occasions where some situation comes up and we need to figure out what we are gong to do with it.

Now before we go into this subject we need to ask an important question and of course this goes back to the things that you heard from Stand French, myself and Bob Dahl in course 3 and that was really asking yourself, " Is it really lost?"

Is the corner really lost? Ken Witt, who passed away a few years ago but who was the Cadastral Chief in Colorado for the BLM, he once said, and it stuck with me every since, he said "when we proportion it is an admission that our profession has failed" meaning that we either haven't found the evidence that is there or the evidence that was left there wasn't substantial enough to

## Is it REALLY lost?

- "Proportioning is an admission that our profession has failed." - ken wit, BLM, Reiried
- We should learn to hate proportioning
- What does it take to be lost?
(We do not proportion lost monuments)
- What do we do with coordinates? remain to our time and so his point being that either way the surveying profession hasn't fulfilled it most basic duty.

So I know I kind of start with that when I have to proportion that I am admitting that my own profession has failed. We should learn to really hate proportioning. That is almost a terrible thing to say anymore because it we are in the big high-tech world where we compute everything and math is easy and technical easy with the

## Restoration of Lost Corners - Part 1

devices and software that we have, but we should learn to hate proportioning. Then another issue here is what does it take for something to be lost? Let's remember that for something to be lost, the evidence has to be gone, there is a little sub comment there we do not proportion lost monuments.

Let's go back and review that for a moment. You will recall that there is a difference between those two terms, corner and monument. A corner is a place, a position, its a location on the surface of the earth. Whereas a monument is a physical object marking that position or al least claiming to mark that position. And here's my point I've been to an awful lot of places where the monument for one reason on another is gone, is lost, but the corner isn't lost but somebody went and proportioned it in anyway, they didn't realize there were bearing trees there, didn't realize there were accessories, other types of evidence, things we discussed in the previous course. So we don't proportion lost monuments. You want to make sure it is really lost that you've really looked for all of the evidence.

Then we have another question that I am gong to ask Ron to address and that is just generically, "What do we do with coordinates?" Coordinates are another form of evidence, but what do we do with those. Ron, you have any comments on that.

Well, I think that you are exactly right, coordinates are another form of evidence. They are measurements. Therefore, we need to evaluate that evidence, the coordinates, and use them accordingly. Sometimes those coordinates are going to be very accurate. Sometimes they are going to be coordinates on the controlling corner plus the lost corner which means it is basically a different measurement of the line. Other times those coordinates are going to be on different positions. They may be coordinates from a position far, far away, or they may be some kind of a local coordinate base that we can't figure out and get on to.

The point is they are evidence and when we are trying to reestablish a lost corner, we want to use the best evidence. So just because it is lost, doesn't mean that we are immediately going to go to proportioning between the nearest found corners. If we have some evidence in the form of coordinates, we may use that. It is evidence that needs to be evaluated. And that's really all it is. I

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think that one of the dangers that we have seen, I know I have seen and you probably have too, is that just because people have some kind of coordinates in there they jump on that real quick and don't even consider the evidence, And you know that may be there.

## Three Corner Conditions

I want to remind you of the three corner conditions which we also reviewed and discussed in course number 3 . There are three basically in the Manual and they are listed here on this slide, the Existent Corners, you find that at 6-11 in the Manual, the Obliterated Corners at 6-17 and then the Lost Corner at 7-2. And what I to remind you is that we when we are talking about proportioning, we are only dealing with the latter of those three. And it takes a lot of effort and energy to get down to the point where it is a lost corner.

Just because it isn't evident the first time you walk up, or whatever, doesn't mean that it is lost and just because someone else didn't find it, doesn't mean that it is lost. What it means is that you still need to do a job to your personal professional satisfaction as to what the evidence situation is there and then therefore which of the three corner conditions you are actually dealing with. So all of that under that question that we asked, "Is it really lost?" So moving on to proportioning itself.

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## General Principles of Proportioning

We have several general principles of proportioning that we want to take a look at. The first one is that proportioning is the last resort. You may have heard people say that and that is true, but the Manual says it too. It doesn't use quick those words, but let's talk a look at it. In 7-1, I am just going to read to you, the rules for the restoration of lost corners should not be applied until all original and collateral evidence has been developed. That goes back with what we just talked about. And when these means have been exhausted.

| General Principles of Proportioning |  |
| :---: | :---: |
|  | There is a specific order <br> for the setting of corner |
| - | (7.7) |
|  |  |
|  | comer |
| line |  |
| 曹 | which will be equitably along the line. |

You might want to underline that word there that is a powerful word, "exhausted", when these means have been exhausted, the surveyor will turn to proportionate measurements. You see, the Manual in its own way is saying that proportioning is the last resort and we need to recognize that and be in harmony with that.

Another principal that you find in 7-5 of the Manual, you don't proportion beyond found evidence. Leave the error where it occurred. Let me read just a couple of sentences here in 7-5. Existing original corners may not be disturbed. Hopefully, we know that.

Consequently, discrepancies between the new measurements and the measurements shown on the record have no effect beyond the identified corners. Now, let's understand something what that means first of all there is really two ways we can look at this.

One is if you've got a record measured relationship between these two existent monuments, anything in between there, that is how you will use that proportioning with the correct method of course, between those two monuments, but if you go on past those to another monument, the difference between those two monuments may be the opposite, it may be longer than the record whereas the first one was shorter than the record. You just leave your proportioning between found existent controlling corners.

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Now the other way I want to look at this is I have know people who for convenience sake have not searched for intervening evidence. In other words, there is a case in California where they found a corner, real rugged country, and the next place that a road crossed that township line was four miles away. So they drove all the way down there; went out there four miles hunted for a day or two; found another corner, real rugged brush country and then used it to do some proportioning in there, but they never looked for the corners in between. So that is another way to look at this.

You can't proportion beyond found evidence. Well you need to look for all the intervening evidence to know what is the found evidence. Because just one corner in between those four miles that comes up and it completely changes the proportion on both sides. So we want to make sure that we are not short changing the process. You know it is not correct to go down to the most convenient next corner and use it to proportion. We need to look at every piece of evidence because the first one you find that's where this "leave the error where it occurred" principal which you see a lot in the court cases comes in.

Another principal here, proportioning gives equal weight to all parts of the line, you'll read that in 7-17. I won't read it to you, it just says that equal relative weight it says and we will see how that plays out here. But one of the things that we want to remember here is that you have to be very cautious and we'll show you a couple of these as we go along where and maybe even unconsciously you find yourself preferring one set of corners over another and a lot of that is just by mistake it's not paying attention to the fact that we need to weigh all parts of the line in a relative way, equally, not picking one set of corners, and saying I am going to force this to be straight, even though other evidence makes it otherwise. And I'll show you some other examples of that as we go along.

Also in 7-7 you will find that there is a specific order for the setting of lost corners. They are listed actually on the top of the next page in Italics. That is another general principal for you. And then finally, one that you don't find in the Manual directly. But I want you to understand the purpose. The whole purpose for proportioning is to compare your chain or whatever your measurement device is, your chain to their chain.

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Here is what you are doing, you are laying them alongside one another. And you are going to find out that your chain is a couple of hundredths of a foot longer than their chain and then you are going to take that difference and apply that in a ratio to everything that is in between those two corners. That's really what proportioning is.

We also need since we are talking about that, we need to maybe think about the mathematical assumption that proportioning makes. In most cases, proportioning assumes that whatever the difference is, the difference is between the record and your measured, that difference is equally distributed along that line. And if you really think about that, that is not always true. That is not always true. I can think of a lot of situations where that is not true.

For instance, maybe you've got a quarter corner lost and you are going to single proportion it in and the east half mile us down in the flats and the west half mile goes up the face of a mountain range, and to say that the difference between the chain and the measurements that they took are equitably or equally distributed along that line is not necessarily true.

Now I am not disagreeing with that principal because that is the foundation of the math in proportioning, but what they should do is scare you a little bit and help you understand why proportioning is the last resort because it is relying on assumptions that are not all that realistic, but that is what happens when you don't have any evidence and you are left with the math and that's all you are left with.

So that is a good reason for us to understand why that is so critical to make sure that it really is lost first of all and that we use the correct method and that we pay close attention to what you are doing when you are comparing those two chains.

## Case Law

Now I just want to mention that we have had some discussion already in the CFedS courses about case law and there is some

## Restoration of Lost Corners - Part 1

case law on this subject, and actually there is quite a bit, we just wanted to throw a couple of things out here for, a brief discussion, if you will, on how the courts have ruled on lost corners.

We have just pulled two cases in here, the first one is an IBLA decision, which you are familiar with now, and there is the number 76-230. It says, "Restoration of a lost corner by proportionate measurement is the proper procedure where no conclusive evidence remains." So you know we have Federal courts and here the IBLA telling us that that is the proper method when no conclusive evidence remains and we have a lot of other cases like that or that say this one as well.

But another one I think is interesting is US versus Doyle, which is a Federal case, and it says and I quote, "Lost corners must be so completely lost that they cannot be replaced by any existing data, all means must be exhausted" and that is probably where the Manual got that term, "to locate original evidence." So hopefully those two just help emphasize what we have been saying in this

## Case Law on Lost Corners

- Brief discussion on how the courts have ruled on "lost" corners
- "Restoration of a lost corner by proportionate measurement....is proper procedure where no conclusive evidence remains..." (IBLA 76-230)
- "Lost corners must be so completely lost that they cannot be replaced by any existing data...all means must be exhausted to locate original evidence". US v. Dovle, 468 F2d 633 (1972)


Now, you know, in proportioning, I know that when I took my state Land Surveying exams and I ended up, well I was licensed in 4 states, but I took 5 exams, because the first time I took the opportunity in Arizona, it didn't work out that well, so I got to take their state test twice. But you know in most states and in most surveyor's minds, there is really only two methods of proportioning in the public land system. We all understand single proportion and double proportion.

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If you take a look at this slide here for a moment, this is something that we kind of drew up just to kind of show you a sample of what types of things are out there, but notice that we not only have a quarter corner that is on a straight line which we are all familiar with but we have a lost section corner at the intersection of 4 lines, which those are the two we are familiar with, but notice that there are a whole lot of other possibilities.

What about a quarter corner with a bearing break on it? What about a lost closing corner up where this section line intersects this military reservation? And what about a lost corner along that military reservation? Also we have a non-rectangular entity, the Elk City townsite and it has a lost corner. What is the proper method for that? And more interestingly, we have a section corner out here where the survey did not continue out this way or this
 way. In other words all we have are these two lines coming together yet it is a section corner. How do we deal with that? And then finally, down here we have a meander corner which is a snubbed out corner. They stopped the survey there and claimed they stopped it there.

So what do we do with that? So you see there are a lot of different possibilities, a lot of different things that we are going to be looking at in the subject of proportioning lost corners. So we are ready to go with the first subject area and that is talking about these different methods.

## Single Proportion

Let's talk about single proportions. Now within the subject of single proportions you might think that this will go fast but no its not going to go very fast because we have 4 different sub methods or categories, not categories really, but 4 different ways to look at single proportions.

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A single proportion is as the Manual talks about is applied to a new measurement made on a line to determine one or more positions on that line, so this is on a line. We find this at 7-16 in the Manual and that was a quote from the Manual so I won't read that one and what it is going to do is it is going to spread the excess or deficiency between the record and the measure along that line at the same ratio all the way across there as the record would indicate and this comes up with a term that I gave you in course number 3, protect the plat meaning we go to the plat and the notes by default and we look at what the record is, what they said they did and then deal with lost corners along the way and those same mathematical ratios.

So a single proportion is simply going to spread out whatever the record measure is, so in other words if you have an extra foot and a half a mile and you are going to put a sixteenth corner in, frankly it will work the same way it goes at midpoint and except in closing sections. Why? Because well first of all it is aliquot on both sides, it is a midpoint corner, so each side of that is going to get an extra half foot. Same thing with a lost quarter corner. So single proportions, at least we start that way with something simple.

Single proportioning is applied to all quarter corners, standard corners, every corner on a standard parallel or correction line, all corners on township and range lines and on straight lines of nonrectangular entities so even an Indian Reservation boundary that is straight in the record for you know eight miles and you've got two or three of the mile corners missing in there the way to properly proportion those is a single proportion.

Well, single proportioning is the only proportionate method that we use to establish corners for the first time. $1 / 16^{\text {th }}$ corners, $1 / 256^{\text {th }}$ corners, $1 / 64^{\text {th }}$ corners, corners along the line, we use a proportionate method, single proportion for and it is the only one that we use that way. Double proportioning we are reestablishing corners, grant boundaries, we are reestablishing corners, but single proportion we actually use to establish corners for the first time. Interesting point.

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## Single Proportion - Midpoint

Let's take a look at a simple single proportion, all right, on your screen we have a quarter corner that is lost on a straight section line. Notice that the bearing of the, the record bearing $89.57 \mathrm{SW}-$ 80.22. Now you could verify this in the notes but that quarter corner was probably set at 40.11 , okay, midpoint in other words. And it is not the 40.11 that we are going to hold, it is the midpoint relationship that we are going to hold because the record said that it was midpoint that both sides of that line was the same length either side of that quarter corner, therefore, we come in we have different measurement.

Notice that our bearing is different. Does that matter? No it doesn't because a line, just a single line, that is all we are talking about was a single proportion, a line is determined by the two points or corners in our case on either end. So the fact that we have a different bearing doesn't change this here. But we do have a different distance. We are at 80.90 . But we know that if you are going to protect the plat and the notes which means that the record says that that was the midpoint and if that quarter corner is truly lost then you are going to set it at 40.45 because that's midpoint based on your measurements. And that is the simplest, that's a midpoint single proportion, those are the simplest ones to do and that is usually what people think about with single proportions.

## Single Proportion - Closing Section

Now let's stay with a quarter corner on a straight line but let's show that we have a slightly different situation when what we call or some of us call closing sections and that is here is a situation where the quarter corner is not at midpoint in the record as many of you know and you should know from what we went through in course 2 , this distance here in a closing section, what is this section six and seven, yeah, this distance here is 40 chains in the record and if it is something different just by chance and I have seen that once in my career, well then fine, just do whatever they indicate.

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I saw one here in Arizona where they midpointed the quarter corner where they weren't supposed to, but that is what they said they did. So that is where we put it back, but that is not normally where that would go. That's 40 and the record here is 79.20 so that means that this is 39 if I can point, 20, right, 20? So we have 40 chains on one side and 39.20 (that's a 9 there) on that side, so we are not at midpoint.

So in order to compute this, we are going to have to set up the actual proportion. Now this is where that analogy of compare your chain to their chain comes in handy. There's what their chain said it was in the record and here is what you say, there's your ratio right there 79.20 is to 79.04 as and you know we could solve for this side, okay, so 79.20 is to 79.04 as 40 is to " X ".

So using that mathematical relationship we solve for " X " and it is just a simple computation solving for one variable, " X " $=39.92$ chains, so what that means is that you would, your distance when your survey is done, you would have 39.92 in this distance and of course, you could compute for this one too, that's a different formula, not a formula, but different numbers that you would put into the computation.

And notice, one of the things that I always try to do to make sure that I did it right, notice that the record is 79.20 and the measure is 79.04 so the measured is shorter so my 40 chains here should be shorter, which it was, 39.92. So you know that is one of the ways to make sure that you didn't do this backwards with the measured over the record or something like that.

And if you wanted to solve this other side, you can actually one of the best ways to do this is a double check and surveyors should be the kings and queens of double checking is I compute what this one is and then I should compute what this one is which is 39.12 and then add them together and it should equal 79.04. So that is one of the ways to always make sure of what you have done, that you did it correctly. So what we have seen so far with single proportions is that both on a straight line, we did the midpoint then we did a non-midpoint.

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Now this leads us into another situation as we pointed out earlier and if you look on this plat. Now if you looked at the original survey of this township, the surveyor said that he was running straight lines down here. But when we get out on the ground, this was done back probably in the ' 70 s or so, we get on the ground we find that he falsified his documents basically. He came from a section corner and went down to his quarter corner and then faked in the notes for the last part.

So when we get on the ground we find that in reality there is these big angle breaks in these quarter corners. Now let's just assume that in 1970 or something, the BLM came in and this is this plat is their survey and you come back in whatever, this is 2007 when we are taping this, and that has become lost. How are you going to put it in knowing that there is an angle break? You know obviously, the last thing you want to do is do a single proportion on a straight line.

You are going to set that corner you know maybe five, six, seven chains away from where it was and our goal is not to put it where is should have been but rather to put it where it was. So you know that is the question, how do we restore those types of situations? So when we look in the Manual and this is at 7-50 and 7-51 in your Manual and you've already been, you've already read chapter 5 at least you were supposed to in preparation for this particular course.

I am just going to read you a little bit here. Sections 7-50 and 751. Some township boundaries not established as straight lines are termed as irregular exteriors. Parts were surveyed from opposite directions and the intermediate portion was completed later by a random and true, leaving a fractional distance. What they are saying is that the township line came in and they stopped and then a couple of years later they came in here and it was over here and so the last half mile has got this angle point in it. So the township line is a single proportion but it's got these two angle points in it. But let's read on.

Such irregularity follows some material departure from the basic rules for establishment of original survey. Sure it does, it was supposed to be a straight line all the way, but now it has a break. But then they say, a modified form of single proportionate

## Restoration of Lost Corners - Part 1

measurement is used in restoring lost corners on such boundaries. Then this last sentence in the paragraph is really the key because that is the situation we were just looking at and it is actually more likely for you and I and that is this is also applicable to a section line or a township line which has been shown to be irregular by a previous retracement and that is the scenario that we laid out.

In 1970, the BLM came in and found that the quarter corners had that big bearing break at them now that quarter corner is lost and our question is, "How do we restore that quarter corner knowing that in reality it was not on a straight line?

## Single Proportion - Angle Break

So if we take a look at this problem that we have here on the screen, we see the situation ere that we have a section corner here and a section corner here, where we have found, all right? And the quarter corner is lost.

Now once again, why would you ever, and believe me I have seen this done quite a few times, the quarter corner was lost and a surveyor comes in and he goes and puts this thing out here. Why would you run this line and put it out here? When we have evidence that that says it was there. Yet that is with blinders on. People aren't realizing that there is a modified form of single proportion when you have an angle break in the record. Now if you were to read the next two or three paragraphs in 7-52 you will be absolutely boggled by what the Manual attempts to tell you to do and I am not going to waste the time to read it you have already read it but I want to explain what it really means.

Understand first of all I call this one of the secret decoder rings of the Manual especially Chapter 7. Understand that most of these methods are written as if you were running record bearing and distance and setting a temporary stake, not something that a lot of modern surveyors do, even in the government. So let's understand that you and I can do that mathematically, going back to the screen, we just made up small numbers here but we have 100 and 100 over here and we just said that's our record and our measured for this example.

Here is the record bearing and distance; here is the record bearing


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and distance. So $f$ you enter those into a COGO program, it is going to give you a record coordinate. A record coordinate over here. And so there's my record and I compare that to my measured. Now if the record and the measured coordinates are the same, that means that you have found the two existent corners in exactly the same relationship in the record as the record said you found them to be in that same relationship. So but that is not likely. They are going to differ. And so this is where we make our adjustment.

Now the way the Manual says to do this if you decode 7-52 (the second half of it), you will find that what it is really saying is that in the direction that the line goes okay so in this case it is an east west one, in the direction that the line goes, we are going to do a single proportion okay which means that we are only going to be using the eastings to help compute that. But for the opposite direction okay, in this case its east west, so for the north south, we are going to use the compass rule. Now that is a bizarre mixture of two different methods and I am not sure that I understand all of the reasoning behind that but the bottom line is that is what the Manual says to do, so we do a single proportion this way to come up with an easting, all right. We will compute an easting off of that but we are going to do a compass rule adjustment for the northing.

## Modified Single Proportion

Now let's take a look at this and solve it then. Let's do the east west first which is a single proportion. If you go back to that page and I'll assume you can do that easier than I can. I've got two dimensions here for the total dimensions between the two points. I have a 81.25 and an 81.33 . So there is my record measured. So what I have done is set up 81.25 is to 81.33 okay there is the ratio. There is laying the two chains down next to each other. And then I am going to go 40.205. Now where did I get that distance from? 40.205? Was that it? I'm sorry, let me look again, yes 40.205. That number comes from the departure of this.

You see if you went north $88^{\circ} 25^{\prime}$ west for 40.22 what that does is it gives you an easting difference here that matches that number. So you see how we are only letting the eastings control this single proportion? So that is where the 40.205 came from. " X " $=$

## Modified Single Proportion

$E / W \cdot \frac{81.25(R)}{81.33(M)}=\frac{40.205}{X} \quad X=40.245(E=59.755)$

N/S - Use Compass rule - Move = South, $0.193(N=100.917)$

If you used compass rule for both dimensions:

Move $=$ S11-34-00W, $0.197 \frac{N 100.917}{E 59.755}$

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40.245. So you would subtract that from the 100 here, the easting 100 , so what that does gives me a easting of 58.756 . Now that is our east west position for this corner. Now we want to do the north south of it. And that is where we use the compass rule.

Now I am not going to go through the math of a compass rule but you should know how to do that you all being licensed surveyors. The compass rule is a basic adjustment method for a lot of different things. In this case we are only going to deal with the north south so if you use the compass rule you are going to move due south .193 that is the southerly move in a compass rule. So that gives us a northing of 100.917 . Now how did I get that? Well because I took the northing in the record and subtracted from it because our move over here is to the south, okay and I subtracted from it the .193 southerly move so that is where I get the 100.917 .

Now here is an interesting thing to look at though, as you see at the bottom of the screen there it says if you used the compass rule for both dimensions, in other words if you didn't do it the way the Manual said but you actually used the compass rule you find out that at least in this case it is the exact same answer. It actually comes out just two or three thousandths of a chain which is not worth worrying about for our survey so it's very interesting that the compass rule comes out so much the same. Now I am just going to tell you what we have shown you here is exactly how to do it by the book. And that doesn't take much time to do and if you are going to use the compass rule instead, in other words you use the compass rule for both dimensions rather than just the opposite dimension.

Recognize that in most cases it comes up the same place or if I could put within a very small amount of difference now what I have found personally and I have kind of played with it over the years is the more the quarter corner is away from midpoint and the more severe that break is, that angular break, the further these two solutions might get. So you know my recommendation is you know if it is relatively flat, which most of them are, relatively flat situation you would probably use the compass rule any way, but you know the only way you are going to know for sure is to do it the correct method and then you will know that you are right on the money.

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Now if somebody else did it previous to you and they used the compass rule, you're probably going to be on their cap anyway but recognize that there are two differences. So that is the modified single proportion and that's for where we have an angle break in the record, all right, for something that would normally have been single proportioned.

Now we have one more category of single proportions and this one is going to take a while and I want to remind you of something in course number 2 which I discussed and I believe may have been mentioned by one of the other speakers too, if you remember a discussion we had in course 2 here with the sample plat about basis of bearings in the Public Land System, and you will recall that the bearing of this west line of the township is north and the bearing of the east line of the township is north and you will recall that in reality those lines are not parallel to each other but because we are on an astronomic basis of bearings, a geodetic system, then what we actually have I exaggerate here but those lines are converging towards each other, right?

And with that in mind then we have to remember that even though we have a bearing here and us regular surveyors are used to working on a rectangular grid, a plane coordinate system, we have to realize that that bearing south $89^{\circ} 56^{\prime}$ east, if it is a continuing bearing that entire six miles than that line is curved. And you will recall discussions in the earlier class that the law said that the north south lines were going to be on the true line, the true meridian, so I'm exaggerating that those are there, and that the east west lines would be 90 degrees to them. So here is exactly what is happening, we got 90 degrees here and we got 90 degrees there and this line has to curve. That is the only way that it can maintain that kind of bearing.


EXERCISE Before moving on, complete the "Irregular Boundary Exercise" which can be found in the Exercise section at the end of this study guide.

diagram A full size version can be found in the Diagram section at the end of this study guide.

## Restoration of Lost Corners - Part 1

## Single Proportions on the Curve

Now I am showing you that to remind you of something we discussed in course 2 because it now affects us in the proportioning of another type of single proportion. So what is happening here and the discussion we are about to have is about single proportions on one of the lines that are curved. Now in the Public Land System there are some folks who might argue a little bit about this, they might expand this list a bit, but the lines that were run as curves in the original survey were the base lines, the standard parallels or correction lines and the north and south boundaries of the township exteriors. They were actually run on a curve.

If you look at this slide, you will see here is the problem. You and I have found point A on the township line and point B which is over here on the township line. All right? And maybe they are four miles apart or whatever. Here's the problem that we have, when you and I are working in this sort of a situation, we are not remembering what the GLO actually did, the General Land Office actually ran this line a tangent to the curve, they may have done it the other way depending upon which way the survey was going, but they actually ran the tangent, that is the line they ran on the ground, but then they computed using the Red Book, they didn't have to compute, they just looked up for what latitude they were at and that told them how far to move each of these corners up to get it on the latitudinal curve.

So in other words the GLO ran the tangent but what they left on the ground is this curve, all right, through here. But here is the issue, when you and I are surveying and we have a coordinate at A and a coordinate at B over here, you and I are in coordinate geometry working on a cord of the curve. In other words, you are not actually on the curve that they ran and that is because you are trying to apply a rectilinear coordinate system to a geodetic coordinate system, and so where we are going with this is Ron is going to discuss and this is where we are really going to get into the discussion on geodesy and basic convergence and curvature in the Public Land System.

Well, next we would like to just take a few minutes and talk about

## Restoration of Lost Corners - Part 1

the geodesy of boundary surveys. And we are not going to get into anything heavy duty but some basic geodesy that affects the boundary surveys. You know in the original surveys, in the oldest ones, they were surveyed with a compass which means they were surveyed as a curved line on the ground because a compass continually is finding the true bearing. The same thing later when they used the solar compass, those were surveyed as a curved line so any east west line was surveyed as a cured line.

Later then we began using tangents and secant lines to deal with that. So what we are going to do is that we are going to take a short break now. When we come back we are going to look at some of the computations, some of the tables, how we can go about dealing with this convergency and the curvature that is involved in a boundary survey on the surface of a curved earth.

So let's take a short break now and when we come back we will deal with that.


EXERCISE
Before moving on to the nex topic, complete the "Single Proportion Exercise" which can be found in the Exercise section at the end of this study guide.


DIAGRAM

TOWNSHIP 15 NORTH, RANGE 20 EAST, OF THE PRINCIPAL MERIDIAN, MONTANA.



## EXERCISE

## Irregular Boundary (Manual Sec. 7-52)



## Calculate N-S

(single proportion for latitude on N -S lines)
1942 latitude S1/2: N. 2708.06
1942 latitude N1/2: N2569.08
1942 total latitude: N.5277.14
Your retracement latitude: N. 5281.07

## Latitude of the S1/2

N. 2708.06 (1942 lat. S1/2) $\div 5277.14$ (1942 total lat.) $=0.513168$
0.513168 x $5281.07($ your retracement lat. $)=\mathbf{N} .2710 .08$ Ft.

## Latitude of the N1/2

N .2569 .08 (1942 lat. $\mathrm{N} 1 / 2$ ) $\div 5277.14$ (1942 total lat.) $=0.486832$
$0.486832 \times 5281.07$ (your retracement lat.) $=\mathbf{N} .2571 .01$ Ft.

## Calculate E-W

(compass rule for departure on $N$-S lines)
1942 dist. of S1/2: 2723.82 ft .
1942 dist. of N1/2: 2581.92 ft .
1942 total dist.: $\quad 5305.74 \mathrm{ft}$.
Difference in departure: 7.08 ft .

## Departure of S1/2

2723.82 ( 1942 dist. $\mathrm{S} 1 / 2$ ) $\div 5305.74$ (total dist.) $=0.513372$
$0.513372 \times 7.08$ (diff. in departure) $=$ E. 3.63 ft . (correction)
E.-292.60 (1942 departure of S1/2) + 3.63 (correction) $=$ E.-288.97 ft.
(departure of this course is minus because it is a NW bearing, the correction is + because it is $E$.)

## Departure of N1/2

2581.92 ( 1942 dist. N1/2) $\div 5305.74$ (total dist.) $=0.486628$
$0.486628 \times 7.08$ (diff. in departure) $=3.46 \mathrm{ft}$.
E. 257.18 (1942 departure of $\mathrm{N} 1 / 2$ ) +3.45 (correction) $=\mathbf{E . 2 6 0 . 6 3 ~ f t ~}$
(the correction is + because it is E.)

Coordinates of the proportioned point: N.12710.10, E.9711.03
Bearing and distance of $S 1 / 2:$ N. $6^{\circ} 05 \prime$ 11" W., 2725.46 ft. ( 41.295 chs.)
Bearing and distance of N1/2: N. $5^{\circ} 47$ ' 19 " E., 2584.16 ft. ( 39.154 chs.)

## SINGLE PROPORTION EXERCISE

| Record | Retrace |  |
| :---: | :---: | :---: |
|  |  | Calculate the position of the $\mathrm{N} 1 / 16 \mathrm{sec}$. cor., $1 / 4 \mathrm{sec}$. cor. and S $1 / 16$ sec. cor. |

Record: North, 79.82
Retrace: N. 5240.25 ft .
W. 8.45 ft .

## Calculate True Bearing and Distance

Dep. $\div$ Lat. $=$ Tan of the bearing
$8.45 \div 5240.25=0.001613$
ArcTan of $0.001613=0^{\circ} 05^{\prime} 33^{\prime \prime}=$ Retrace bearing: N. $0^{\circ} 05^{\prime} 33^{\prime \prime} \mathrm{W}$.
Dep. $\div \sin$ of the bearing $=$ Dist.
$8.45 \div 0.001613=5240.26$

## Calculate Proportion

Retrace distance $\div$ Record distance $=\mathrm{K}$
$5240.26 \div 5268.12=0.994712$
K x record dist. $=$ Proportionate dist.

$$
\begin{array}{rlrl}
0.994712 & \mathrm{x} & 1320.00 & =1313.02 \\
\text { " } & \mathrm{x} & 1320.00 & =1313.02 \\
\text { " } & \mathrm{x} & 1320.00 & =1313.02 \\
" \quad & \mathrm{x} & 1308.12 & =\underline{1301.20} 5240.26
\end{array}
$$

## Proportionate Position of the Corners

$\operatorname{Sin} 0^{\circ} 05^{\prime} 33^{\prime \prime} \times 1313.02=2.12$
$\operatorname{Cos} 0^{\circ} 05^{\prime} 33^{\prime}$ x $1313.02=1313.02$
" $\quad$ x $1313.02=2.12$
" $\mathrm{x} 1313.02=1313.02$
" $\quad$ x $1313.02=2.12$
" $\quad \mathrm{x} \quad 1313.02=1313.02$
" $\quad$ x $1301.20=2.10$
" $\quad \mathrm{x} \quad 1301.20=1301.20$

S 1/16: N. $1000.00+1313.02=\mathbf{N} .2313 .02$
E. $1000.00-2.12=$ E. 997.88
N. 1/16: N. $3626.04+1313.02=\mathbf{N} .4939 .06$
E. 995.76-2.12 = E. 993.64
¼ Sec. Cor.: N. $2313.02+1313.02=$ N. 3626.04
E. $997.88-2.12=$ E. 995.76

Sec. Cor.: N. $4939.06+1301.20=$ N. 6240.26
E. 993.64 - $2.10=$ E. 991.54

## Course 4: Restoration of Lost Corners Study Guide

## COURSE DESCRIPTION:

## COURSE

 OBJECTIVES:
## COURSE INSTRUCTOR(S):

VIDEO LECTURE TITLE:

This course consists of four videos, some reading, and three exercises, on the "Restoration of Lost Corners". The legal, mathematical, and practical applications of the methods of proportioning, as found in the Manual of Surveying Instructions, are presented. Students will be able to address what corners control in most situations, how to proportion properly, what legal principles are involved when proportioning, and how to deal with the latitudinal curve. A lengthy discussion of convergence and curvature in the PLSS is also included.

Upon completion of this course, students will be able to:

- Define the three corner conditions listed in the Manual of Surveying Instructions
- Describe, identify applicability, and compute proportions using all methods
- Demonstrate an understanding of curvature in the PLSS

Dennis Mouland, Bureau of Land Management
Ron Scherler, Bureau of Land Management

Restoration of Lost Corners - Part 2 (42 minutes)
WEB COURSE

## Restoration of Lost Corners - Part 2

## Introduction

Welcome back. First of all I would just like to look at a couple of principles, a couple of issues we have to deal with when we are working on the surface of the earth when we are trying to work with true bearings.

And I think these will help us maybe understand exactly what is going on here.

Now first of all a straight line has a constantly changing bearing unless it is exactly north south or of course if its on the equator. But a straight line has a constantly changing bearing, it is a curved line as far as bearing goes.

It is a straight line, even if we have line of site, a straight line the bearing is constantly changing unless it is exactly north south or at the equator.

A line of constant bearing is a curved line, unless it is exactly north south. So if we want a line of constant bearing, which is what we find in the Public Land System, if we want that, it is a curved line. If we have a straight line it is a line of constantly changing bearing. That is a real key issue here.

An east west line has the most curvature or change in bearing as it approaches north south, it has less and less and less change because it is a function of its departure.

Lines approaching north south have almost no curvature or change in bearing because there is very little departure and it is a function of departure.

## Geodesy for Boundary Surveys

- A straight line has a constantly changing bearing unless it is exactly North or South
- A line of constant bearing is a curved line unless it is exactly North or South.
- An East-West line has the most curvature or change in bearing
- A line approaching North-South has almost no curvature or change in bearina


## Restoration of Lost Corners - Part 2

First of all let's just look at a very simple example and see how we work through and deal with this curvature, how we get a true bearing of a line on the surface of the earth and how that affects our corners, how that affects those things.

First of all let's look at this one, Latitude 44 degrees of course the closer we are to the equator, the less change there is, the less convergency, the less curvature, the closer we are to the pole, the more curvature, the more convergency. True bearing. How do we come up with the true bearing here?

So let's assume that over here at point A that line to the north is true north. We have that bearing, we've established that, we have turned an angle and we have a distance. So with that angle we can compute that our forward bearing at least at point A is south $89^{\circ}$


## Restoration of Lost Corners - Part 2

Let's say that we can sit with our instrument on point A and we can see point B so we know that it is a straight line, we know what the forward bearing is and now we know what the departure of that line is.

So we go to the standard field tables and that is contained in your resource disc in your set of DVDs, the standard field tables, a lot of interesting information in there and where we want to go is to Table 11, and Table 11 is convergency of meridians, six miles and six miles apart and differences of latitude and longitude.

Well, meridians six miles long and six miles apart, well what we are talking about here is a township really and that helps us think about what this table is giving us actually It is talking about a township. And it gives us various pieces of information about this township for various latitudes.

It gives us convergency, difference in difference in longitude difference in latitude and the convergency is on the parallel or on the angle.

So let's go and look at our table for the situation that we are dealing with which is 44 degrees. So at 44 degrees, we are going to go to our table, we are going to go to our table and right here we are going to find that the convergency is 70.1, but that is not really what we are looking for that is talking about linear convergences.

So we want to go to the next table, the next line and it tells us that the meridians which are six miles apart remember, they converge zero degrees five minutes one second at this latitude, at 44 degrees, meridians that are six miles apart are going to be converging by five minutes one second.

## Restoration of Lost Corners - Part 2

So what does that tell us. Table 11 tells us that the bearing of a straight line $A B$ is going to change zero degrees five minutes and one second in six miles.

Because those meridians are converging. So our straight line bearing is going to change five minutes and one second in six miles. Well, so how much will it change in 232.44 chains because that is the departure of our line $A B$.

Well, if we know it is going to change five minutes and one second in 480 chains, that is six miles, 480 chains equals six miles and these meridians are six miles apart in our table, so we divide five minutes and one second, that is 301 seconds by 480 chains and that gives us the factor .62708 seconds per chain so that is how much our bearing is going to change per chain of departure.
 south west bearings the correction is applied counter clockwise. So if you have a line that is exactly west, north west or south west, the correction is applied counter clockwise.

If you have north east or south east bearings, the correction is applied clockwise and that includes bearings that are exactly east. So any line that is going easterly to any extent, it is clockwise. If the line is westerly, it is counter clockwise.

## Restoration of Lost Corners - Part 2

Now another way to look at it is if it is a northerly bearing, north east, north west, the correction applied is added. If it is south east or south west, the correction is subtracted and if it is east or west exactly, the correction is subtracted. Just another way to look at it.

We have a couple of rules there to help remember which way we are going to apply this correction. Because it can get confusing, but once you get it in your head exactly what is happening, you kind of get a picture of it, it is much easier to apply that rule or that correction in the proper method.

So if we look at our diagram, we can see south $89^{\circ} 44^{\prime} 40^{\prime \prime}$ west minus because what do we have, we have a south west bearing, it is a south one so we subtract it or it is a westerly one so we go counter clockwise. So either rule works here. It is just however you want to look at it.

So back to our diagram, we can see that $89^{\circ} 44^{\prime} 40^{\prime \prime}$ south west minus 0.0225 seconds that's our correction equals a bearing of south west or since it is a back bearing, now it is north east $89^{\circ}$ $42^{\prime} 15^{\prime \prime}$. That is our back bearing. We now have a straight line between points.

We have a forward bearing and a back bearing, so it is a simple matter to get a mean bearing. And that comes about by just averaging those two and we end up with a mean bearing of $89^{\circ}$ 43 ' $27.5^{\prime \prime}$. So what is that telling us?

That is telling us that when we establish corners on this line, this line is going to have several quarter corners and section corners in this 232 chains that we can't put them on a straight line because they were established on a true bearing across there. We need to establish them on this curved line.

They need to be on a line that has a constant bearing of $89^{\circ}$ $43^{\prime} 27.5^{\prime \prime}$ and as we discussed earlier, a line that has a constant bearing is not a straight line it is a curved line. So we need to find a way to do that and I am going to show a couple of examples, two different ways, and there are actually others.

One way to go about this is to run a tangent and the Manual talks

## Restoration of Lost Corners - Part 2

about running a tangent line and the Manual talks about running a secant line. There are several ways to go about this. I am going to give you a couple of examples of how to account for curvature in your calculations and I think these work pretty well for your day to day surveying.

Also of course there are also programs out there that take care of geodesy and deal with the curvature that you may be familiar with and that you may be using. But it is important that we have a good understanding of what is actually happening so that we can catch errors, so we make sure that we do it right and we really know what is going on.

So let's look at this and let's assume that point $\mathrm{a}, \mathrm{b}$, the small a , small $a, b, c$. d and e are temporary points.

So this is a situation, this is probably something that is going to happen maybe you are surveying in the northern California coast and it is pretty brushy, pretty nasty country and you have to actually survey this on the ground with a traverse on the ground, so you are setting a temporary point somewhere near where you believe the corner is going to end up so you have a point at 3985 5 , one at 80.024 , one at 120.107 and so forth.

Now for simplicity I have these all on a straight line, these are all on a straight line. Most of the time that is not going to happen, you are going to have some bends. But I think you will see that it's the same process is used even if you have several traverse points between these temporary points.

So we have a total distance of 232.45 . We' re at latitudes 44 degrees and we have these temporary points, we have a forward bearing. We have a back bearing. So let's see what we can do here. Well, first of all remember we had this factor that we got by dividing five minutes and one second which is the curvature and meridian six miles apart at 44 degrees, it comes from Table 11 in the standard field tables.

We divide that by 480 chains, again which is six miles in chains and we come up with a factor, curvature per chain in seconds 0.62708 . Well, let's look at this first course. If we want to compute the true bearing of this straight line at point a we can take

## Restoration of Lost Corners - Part 2

0.62708 our factor per chain times the departure in chains between point a (small a) that's the departure and we end up with 24.9923 seconds.

So in that half a mile basically, this line, this straight line of ours, the bearing is going to change 24.9923 and we just move on down here to the next point the bearing is going to change 50.1808 .

At the next point the true bearing of the line by the time we get to point c on our straight line, the true bearing has changed one minute 15.3161 seconds. So we can see as we go across that bearing keeps changing. We keep going you know to point $d$ and finally over here at point e . The departure there is two minutes .5151 seconds. So now we know what the correction or the curvature of that line, the change in bearing of that line is as we go across.

Beginning with our forward bearing of $89^{\circ} 44^{\prime} 40^{\prime \prime}$, we know that at point a , that bearing has changed 24 seconds, at point b it has


## Restoration of Lost Corners - Part 2

If we have the bearing at each point, all we have to do is mean them to get the mean bearing of the line. So the mean bearing of line Aa is $89^{\circ} 44^{\prime} 27.5^{\prime \prime}, \mathrm{Ab}$ is $89^{\circ} 44^{\prime} 02^{\prime \prime}, \mathrm{Bc}$ is $89^{\circ} 43^{\prime} 37^{\prime \prime}$ and so on. Now what do we have here? We have temporary points that we have established.

We surveyed them on a straight line. We have now computed true bearing between each of those points. So we have a mean true bearing for each course across this traverse, between these points. You are done. That is all you need to do to account for convergency.

From this point you can completely forget about the convergency. All you need to do is use the bearings and distances that you have computed now and you can compute your corner moves the way you always do. You can do your proportionings the way you always do.

From now on once you get to this point you can forget about convergency. It is accounted for it. You've accounted for it when you put the mean bearings on each of these courses. You've created a curved line across there.

Now as we'll talk about in a little bit. There is one place that you have to still think about it and that is the linear convergency because any time we use true bearing, figures do not close, we'll get back to that in a minute but for this calculation if you were going to single proportion all the corners along this line, let's say if this is the south boundary of a township and we have these five missing corners and we are single proportioning these five missing corners, you have taken into account convergency.

All you have to do now is a single proportion using these bearings and distances and you are done. It is taken care of. All right, so let's take a look at a different method now.

So if we have a single proportion along this line and we need to reestablish these five lost corners, we have already accounted for convergency, we have accounted for the curvature I mean, and all we need to do is a normal single proportion using the numbers that we get from these bearings and distances. It is done. You have taken care of it.

## Restoration of Lost Corners - Part 2

You have a mean bearing now of $89^{\circ} 43 \prime 27.5^{\prime \prime}$ for the entire line. That is our mean bearing and I just wanted to show exactly what is happening here. The straight line, it is straight but its bearing is changing and that is what is represented by the line that I just put on here.

It says it is a straight line, but actually it is but its bearing is changing, its bearing is curving. Its bearing off this way is changing counter clockwise. It is rotating counter clockwise. I hope this picture helps. Picture in your mind a little bit what is happening there. Now I want to look at another example which is a different method for computing this and this would be a situation where we have coordinates on either end of the line.

So here we have coordinates on each end we do not have temporary point in the middle. And what we are trying to do is we are trying to calculate the position of each of these lost corners along this line and put each of these corners on the curve and again, we have the same forward bearing and back bearing, we've computed the mean bearing, we got the same distance and we have a computed departure here. So let's see how this method changes a little bit and there are a couple of things that are different and they are important differences.

First of all, we have to proportion. Right? Because we are doing a single proportion here so we have 232.447 chains. Remember that was the departure of this whole course, so we are going to divide that by six, because we have six equal segments across here. Six half miles between original corners and it equals 38.7412 chains per half mile.

These originally were 40 so there is some error in here. So now we know that we want to establish a point, we want to begin at point a and we want to go at a mean bearing of $89^{\circ} 43^{\prime} 27.5^{\prime \prime}$ that is our bearing south west 38.7412 chains. So how do we do that? Well, we need a point at each of these.

Let's start by same thing, zero degrees five minutes one second divided by the 480 chains gives us the .62708 . That is our change
 per chain. We need that again. So now we want to calculate the true mean bearing Aa and that is not exactly what we are doing but

## Restoration of Lost Corners - Part 2

let's think about this.
We have already computed what the mean bearing of this entire line is so corners $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}$ and e all want to be on that mean bearing. So we are going to take half of line Aa times our factor and that is going to give us the correction we need to get from a forward bearing to a bearing on the true mean bearing, which is the curved line. So from the forward bearing to the mean bearing on line Aa ends up being 12.1469 seconds.

Now we used half of the distance to the line because we are not trying to calculate how much that line the bearing changes in that half mile, we are trying to calculate the mean of that line. So we only want the chain for half of it. At the midpoint. That is how we are going to get the mean. So this is the change in bearing of a straight line Aa. And what this actually is is it is the angular difference between our mean true bearing and a forward bearing to point A. And we will look at that again a little bit more so that we are certain that everyone understands that. So let's look at the next one, b, 24 seconds and again we go 38.7412 chains. That is the midpoint of line Ab. Capital Ab. The midpoint of that line times our factor gives us 24.2938 seconds. That is the angular change between the true bearing which we know, remember we calculated the true mean bearing of this line, $89^{\circ} 43^{\prime} 27.5^{\prime \prime}$ seconds.

We want to go from that true mean bearing to a forward bearing that will take us from point A, Capital A, to point b. Let's look at the next one. Again, midpoint of line Ac times our factor gives us 36.4407 seconds. That tells us how much curvature we've got across there and we are trying to get now a forward line from A to C.

We already know what the true mean bearing is, the curved line, we are trying to figure out what angle do we need to turn now to get a forward bearing that will take us here and it is it's going to change 36.4407 seconds. And you will see how this works as we go along. We go on for point Ad, Ae, same process and the last one clear across.

## Restoration of Lost Corners - Part 2

Now let's look at the next step and see what happens. First of all which way are we going to apply this. Now remember that when we had a forward bearing, a straight line and we calculated this factor, this curvature for each segment of the line, we said that we applied it counter clockwise, if it was a westerly bearing and clockwise if it was an easterly bearing. In this case, we are not going from the straight line to the true mean bearing.

We are going from the true mean bearing back to a forward bearing, the straight line. We are going in the opposite direction. We're going to use these numbers that we created to go from the true mean bearing to a forward bearing so that you can calculate where these points go from your coordinates. So let's look at what happens.

Because we are laying out a line of constant bearing, we begin with a true bearing and apply the corrections to get to the forward bearings from point A to each of the corner points. That is an important factor. So we are going to apply these corrections in the opposite direction. We are doing a different thing. We are laying out a line here. We are laying out a line.

In our previous example, we had points on the ground that we had measured between and we were trying to calculate what the mean bearing was through it. Here we are doing the opposite.
Clockwise for westerly. Counter clockwise for easterly. The opposite of the example before. So let's look at how this works. Again here are our coordinates, here is our diagram, forward bearing, back bearing, mean bearing.

## Restoration of Lost Corners - Part 2

Here's what happens. Again our proportions 38.7417 . Here's our corners. Now south $89^{\circ} 43^{\prime} 27.5^{\prime \prime}$ that is our true mean bearing. So what we want to do is remember we are going south west, so we are going to go clockwise. We are going to add our correction which is 12.1469 seconds and that gives us a forward bearing from point A of $89^{\circ} 43^{\prime} 39.6469^{\prime \prime}$.

That is what we need to lay off on the ground at point A we need to turn an angle from that north line that will give us $89^{\circ} 43^{\prime} 39.6469^{\prime \prime}$. That is the forward bearing to get us to point A. From Ab we do the same thing except that now we add the 24 seconds. And you notice we get a bearing of $89^{\circ} 43^{\prime} 51^{\prime \prime}$. That is the forward bearing at point A that will give us this corner point, this true corner point for $b$.

Where we want to set the corner. This proportion position. We do the same thing for c . We add the factor that gives us a new bearing.

I want you to see what happens now as we continue across here and we go $\mathrm{a}, \mathrm{b} \mathrm{c}, \mathrm{d}$ and e and on across. Now we will go to this next slide and we will see. Watch what happens with line Ad. Again we add the correction.

We add it to the mean bearing and that gives us a forward bearing to get from A to d and we do the same thing with Ae. Again we add it. Look what happens. When we add that correction from the mean bearing we add the correction and we get south $89^{\circ}$ $44^{\prime} 40.3815^{\prime \prime}$ seconds west. Look what our forward bearing was to begin with $89^{\circ} 44^{\prime} 40^{\prime \prime}$.

Of course we rounded it off but we match. That is the same bearing. All we have done is we have computed the opposite direction now. We have computed from the mean bearing to get
 to the forward bearing so that we can lay this off because what we were doing before we had already set monuments and we were working to compute where those temporary corner points were.

Now we were trying to compute where to set those points and of course in this case, they are not temporary, we have computed where the true corner point goes. Where the proportion corner

## Restoration of Lost Corners - Part 2

point goes is on these forward bearings and at the correct distance.
So to get the coordinates let's just work through one example to get the coordinates of point c . What would we do? Well, we know the departure of $c$ is 116.2237 , you can check back and you will see that that is correct. We know the sign of our forward bearing that we computed is $89^{\circ} 44^{\prime} 04^{\prime \prime}$, we know that that is the forward bearing so we divide the departure by the sine of the bearing and that gives us the distance of Ac which comes up right here 116.2249 very little change.

So then we can take the co sine of the bearing times the distance and that gives us the north or south component, here we have a south west line so it gives us the southing of that line basically.

We take the sign of the bearing times the distance, the length of the line and that gives us the west component. So now we know from point A this is the coordinate that we want to set our monument and we have taken care of the convergence here, and now we get to the coordinate, we have to add those of course to our initial coordinate, add or subtract, and we do that and we end up with the coordinates of point c . We don't have to think about curvature anymore. We have taken care of curvature now.

Now we are just back to just surveying plain. We have computed these forward bearings, all we have to do is turn these forward bearings. We have got our bearings at A established from point A we have computed forward bearings and distances to each of these points.

We can compute coordinates on each of these points and we can go out on the ground and set the monument at those points. And they will be on a curve. They will not be on a straight line. They will be on a line of constant bearing on a straight line. Two different methods, one when we have set temporary points to begin with.

How we deal with curvature. Number 2 when we have coordinates on the controlling corners and we need to calculate corner points within. Pretty basic. It is not high level geodesy. But it is well within the accuracy that we need for these boundary surveys. It is pretty straight forward.

## Restoration of Lost Corners - Part 2

It is not very difficult to really grasp once you work with it a little bit. To begin with it is a little bit confusing because straight lines aren't really straight and curved lines aren't really curved and all that kind of thing. But once you work with it a little bit, I think that you will see that either one of these methods are pretty easy to work with and it is pretty easy to program a calculator to do it and if you are using some other kind of program that already has curvature and geodesy built into it. Wonderful. Hopefully, this just helps to understand exactly what is going on.

Now there is one more thing that we need to look at and that is linear convergency because we know that if we use true bearings and distances to calculate around a closed figure, it is not going to close, even if all of our measurements and bearings and angles are exact.

It will not close because meridians converge. So we need to talk about that and we have to account for any time we are using true bearings.

So the linear convergency we have to deal with and we will just look at an example here and it is a function of latitude okay because we have more convergency of meridians the further north we go so it is a function of latitude but it is also a function of area.

It doesn't really matter what shape we are dealing with, it is area. So it doesn't matter if we have a shape that is six miles by one mile or if we have a shape like this red figure here that contains six square miles also each of these areas contain six square miles therefore, and they are at the same latitude, therefore they are going to have the same linear convergency.


## Restoration of Lost Corners - Part 2

Now from the standard field tables, again we go back to the standard field tables, and you know it is six miles apart again. We go over to this same table and we look at our 44 degrees again and this time we are going to look at the first line which is 70.1 links. And what that tells us is that in a township six miles by six miles.

If we survey around the exterior of that township and it is exactly six miles by six miles and all of those bearings are north south east west lines and we report true bearing on all of those lines, that township then will misclose by 70.1 links. It's telling us that in a township those meridians are converging by 70.1 links in that area in 36 square miles those are going to converge.

So we can work with that to calculate the convergency for any area of that township. So let's go back and look at our example
 and we will do this one at 48 degrees so linear convergency for a figure that has six square miles at 48 degrees. If we go to the table we will find that a township at 48 degrees has 80.6 links of linear convergency.

So it is a pretty simple matter to divide that by 36 square miles that gives us 2.2389 links per square mile and of course if we wanted to compute this factor in square feet or square chains, you know we could do it in anything just by what we divide it. So we've got 2.2389 links per square mile times six square miles gives us 13.43 links so at 48 degrees any figure, any closed figure that contains six square miles, doesn't matter the shape, doesn't matter, nothing else, just 48 degrees six square miles.

If we used true bearings all the way around it and we computed closure, its going to misclose by 13.43 links and what happens is if we go around clockwise, if we are calculating our misclosure clockwise, we are not going to have enough easting that's what is going to be short.

There is going to be more westing than there is easting because as we head north the meridians converge. So 13.43. And I just wanted to show you one other thing. If we look at our table again and we go to 48 degrees and we'll see that at 48 degrees there's 5.46 seconds of convergency in those meridians that are six miles apart.

## Restoration of Lost Corners - Part 2

Well, that means that if we started over there at the north east corner and we are now over at the north west corner or the south west corner, our bearing of this line six miles away is going to change by 5.46 seconds. That means that the bearing of the west boundary of this figure is 5 minutes and 46 seconds north west, it's not really north if we turn 90 degree angles. Well if we calculate, if we take the sign at 5 degrees 46 minutes times 80 degrees or 80 chains, we end up with 0.134 chains, 13.4 links, the same thing. It is a result of the convergence of the meridians is why we have that linear convergency.

Why it does not close and any time we are using true bearing, it is important that we take that into account. Now this has been a pretty, I guess a pretty quick review of what is going on with convergency and with curvature, with how it has an effect on what we do in the public land survey and how we need to apply it when we are doing our calculations for a single proportion when we are dealing with a range line or a township boundary, south boundary, north boundary of the township, any major east west line that has curvature in it, how we have to deal with it, how we can compute it, how we can account for that in our calculations, and put our corners when we are reestablishing corners back on the curve, how we can compute bearings and then how we need to deal with that linear convergency when we are all done to make sure that our figure really does close and our work is really accurate.

So I hope that this gets you along the way to understanding that. Of course in your material there is a good paper about curvature and convergency that I hope you read and then of course there is an exercise to kind of practice these skills and I hope that will end up being beneficial too. So hope that this is helpful we'll go on to the remainder of our discussion of the restoration of lost corners now.


Exercise Before moving on to the next topic, complete the "Single Proportion with Curvature Exercise" which can be found in the Exercise section at the end of this study guide.


PRoвLEM Before moving on to the next topic, complete the Problem "Mean Bearing \& Linear Convergency" which you can access from the course description page.
handout The paper on Curvature and Convergency can be found in the Handout section at the end of this study guide.

## EXERCISE

## Single Proportion with Curvature



## Calculate the True Mean Bearing of the Line

Calculate forward bearing
Forward bearing at A (cor. of secs. 5, 6, 31 and 3)2: N. $0^{\circ} 00^{\prime} 00^{\prime \prime}$ E. $+89^{\circ} 44^{\prime} 19^{\prime \prime}=\mathbf{N . ~ 8 9}{ }^{\circ} \mathbf{4 4} \mathbf{1 9}^{\prime \prime} \mathbf{E}$.

## Calculate angular convergence:

Angular convergence for meridians 6 miles apart at Lat. $47^{\circ} 30^{\prime}$ :
From Table 11: $\quad$ Convergence at $47^{\circ}=0^{\circ} 05^{\prime} 34^{\prime \prime}$
Convergence at $48^{\circ}=0^{\circ} 05^{\prime} 46^{\prime \prime}$
Convergence at $47^{\circ} 30^{\prime}:\left(0^{\circ} 05^{\prime} 34^{\prime \prime}+0^{\circ} 05^{\prime} 46^{\prime \prime}\right) \div 2=0^{\circ} 05^{\prime} 40^{\prime \prime}$
$0^{\circ} 05^{\prime} 40^{\prime \prime}=340^{\prime \prime}$
$340^{\prime \prime} \div 31680.00 \mathrm{ft} .(6 \mathrm{miles})=0.010732^{\prime \prime}$ per ft . of departure
0.010732 " x 7960.73 ft . ( $1 / 2$ departure of the line) $=0^{\circ} 1^{\prime} 25.44$ "

Mean bearing: N. $89^{\circ} 44^{\prime} 19^{\prime \prime}$ E. (forward bearing) $+0^{\circ} 1^{\prime} 25.44 "($ correction $)=\mathbf{N . ~ 8 9}{ }^{\circ} \mathbf{4 5}, \mathbf{4 4 . 4 4 "} \mathbf{E}$.
(The correction is applied clockwise because the bearing is easterly and we are going from forward bearing to true bearing)

Calculate departure of the line
Departure of the line: $\operatorname{Sin} 89^{\circ} 44^{\prime} 19$ x $15921.62=$ E. 15921.45 ft .

## Calculate the single proportion

The record calls for 6 equal 40 ch . segments, therefore: E. $15921.45 \div 6$ = E.2653.58 ft.

## Calculate the forward bearing from $A$ to each of the lost corners

## Line A-B:

E. $2653.58 \div 2=1326.79 \mathrm{ft}$. ( $1 / 2$ the departure of line A-B)
E. $1326.79 \times 0.010732$ " (angular convergence per ft. of departure) $=0^{\circ} 00^{\prime} 14.24^{\prime \prime}$ (angular convergence)
N. $89^{\circ} 45^{\prime} 44.44^{\prime \prime} \mathrm{E}$. (mean bearing line A-G) - $0^{\circ} 00^{\prime} 14.24 "\left(\right.$ angular convergence) $=\mathbf{N} . \mathbf{8 9}^{\circ} \mathbf{4 5} \mathbf{3 0 . 2 4 " \mathbf { E }}$. (We are going from true bearing to forward bearing therefore the correction is applied counterclockwise for easterly lines and clockwise for westerly line. The opposite is true when going from forward bearing to true bearing.)
Distance of line A-B: 2653.58 ft . (departure of A-B) $\div \sin 89^{\circ} 45^{\prime} 30.24^{\prime \prime}=\mathbf{2 6 5 3 . 6 0} \mathbf{f t}$.
At Pt. A the forward bearing and distance to Pt. $B$ on the curve is:
N. 89 ${ }^{\circ} 45$ ' 30.24" E., 2653.60 ft

Coordinates of the proportioned point: N.10011.19, E.12653.58

## Line A-C:

E. $5307.16 \div 2=2653.58 \mathrm{ft}$. ( $1 / 2$ the departure of line A-C)
E. $2653.58 \times 0.010732$ " (angular convergence per ft. of departure) $=0^{\circ} 00^{\prime} 28.48^{\prime \prime}$ (angular convergence)
N. $89^{\circ} 45^{\prime} 44.44^{\prime \prime} \mathrm{E}$. (mean bearing line A-G) - $0^{\circ} 00^{\prime} 28.48^{\prime \prime}($ angular convergence $)=\mathbf{N} . \mathbf{8 9}^{\circ} \mathbf{4 5}{ }^{\prime} \mathbf{1 5 . 9 6 "} \mathbf{E}$.
(We are going from true bearing to forward bearing therefore the correction is applied counterclockwise for easterly lines and clockwise for westerly line. The opposite is true when going from forward bearing to true bearing.)
Distance of line A-C: 5307.16 ft . (departure of A-C) $\div \sin 89^{\circ} 45^{\prime} 15.96^{\prime \prime}=\mathbf{5 3 0 7 . 2 1} \mathbf{~ f t}$.
At Pt. A the forward bearing and distance to Pt. C on the curve is:
N. 89 ${ }^{\circ}$ 45' 15.96 " E., 5307.21 ft

Coordinates of the proportioned point: N.10022.75, E.15307.16

## Line A-D:

E. $7960.74 \div 2=3980.37 \mathrm{ft}$. ( $1 / 2$ the departure of line A-D)
E. $3980.37 \times 0.010732^{\prime \prime}$ (angular convergence per ft. of departure) $=0^{\circ} 00^{\prime} 42.72^{\prime \prime}$ (angular convergence) N. $89^{\circ} 45^{\prime} 44.44^{\prime \prime}$ E. (mean bearing line A-G) - $0^{\circ} 00^{\prime} 42.72$ " (angular convergence) $=\mathbf{N} . \mathbf{8 9}^{\circ} \mathbf{4 5}$, 01.72" $\mathbf{E}$. (We are going from true bearing to forward bearing therefore the correction is applied counterclockwise for easterly lines and clockwise for westerly line. The opposite is true when going from forward bearing to true bearing.)
Distance of line A-D: 7960.74 ft. (departure of A-D) $\div \sin 89^{\circ} 45^{\prime} 01.72^{\prime \prime}=\mathbf{7 9 6 0 . 8 1} \mathbf{f t}$.
At Pt. A, the forward bearing and distance to Pt. $D$ on the curve is:
N. $89^{\circ}{ }^{45}$, 01.72" E., 7960.81 ft.

Coordinates of the proportioned point: N.10034.67, E.17960.74

## Line A-E:

E. $10614.32 \div 2=5307.16 \mathrm{ft}$. ( $1 / 2$ the departure of line A-E)
E. $5307.16 \times 0.010732$ " (angular convergence per ft . of departure) $=0^{\circ} 00^{\prime} 56.96^{\prime \prime}$ (angular convergence) N. $89^{\circ} 45^{\prime} 44.44^{\prime \prime} \mathrm{E}$. (mean bearing line A-G) - $0^{\circ} 00^{\prime} 56.96 "\left(\right.$ angular convergence) $=\mathbf{N} . \mathbf{8 9}^{\circ} \mathbf{4 4}{ }^{\prime} \mathbf{4 7 . 4 8 "} \mathbf{E}$. (We are going from true bearing to forward bearing therefore the correction is applied counterclockwise for easterly lines and clockwise for westerly line. The opposite is true when going from forward bearing to true bearing.)
Distance of line A-E: 10614.32 ft . (departure of A-E) $\div \sin 89^{\circ} 44^{\prime} 47.48^{\prime \prime}=\mathbf{1 0 6 1 4 . 4 2} \mathbf{~ f t}$.
At Pt. A, the forward bearing and distance to Pt . E on the curve is:
N. 89 ${ }^{\circ}$ 44' 47.48 " E., 10614.42 ft.

## Coordinates of the proportioned point: N.10046.96, E.20614.32

## Line A-F:

E. $13267.90 \div 2=6633.95 \mathrm{ft}$. ( $1 / 2$ the departure of line A-F)
E. $6633.95 \times 0.010732^{\prime \prime}$ (angular convergence per ft . of departure) $=0^{\circ} 01^{\prime} 11.20^{\prime \prime}$ (angular convergence)
N. $89^{\circ} 45^{\prime} 44.44^{\prime \prime}$ E. (mean bearing line A-G) - $0^{\circ} 01^{\prime} 11.2$ " (angular convergence) $=\mathbf{N} .89^{\circ} \mathbf{4 4}, \mathbf{3 3 . 2 4 "} \mathbf{E}$. (We are going from true bearing to forward bearing therefore the correction is applied counterclockwise for easterly lines and clockwise for westerly line. The opposite is true when going from forward bearing to true bearing.)

Distance of line A-F: 13267.90 ft . (departure of A-F) $\div \sin 89^{\circ} 44^{\prime} 33.24^{\prime \prime}=\mathbf{1 3 2 6 8 . 0 3} \mathbf{~ f t}$.
At Pt. A, the forward bearing and distance to Pt. F on the curve is:
N. $89^{\circ}{ }^{\circ} 44^{\prime} 33.24$ " E., 13268.03 ft.

Coordinates of the proportioned point: N.10059.61, E.23267.90

## Line A-G:

E. $15921.48 \div 2=7960.74 \mathrm{ft}$. ( $1 / 2$ the departure of line A-G)
E. $7960.74 \times 0.010732 "$ (angular convergence per ft. of departure) $=0^{\circ} 01^{\prime} 25.43 "$ (angular convergence)
N. $89^{\circ} 45^{\prime} 44.44 "$ E. (mean bearing line A-G) - $0^{\circ} 01^{\prime} 25.43^{\prime \prime}\left(\right.$ angular convergence) $=\mathbf{N} . \mathbf{8 9}^{\circ} 44 ’ \mathbf{1 9 . 0 1 "} \mathbf{E}$. (We are going from true bearing to forward bearing therefore the correction is applied counterclockwise for easterly lines and clockwise for westerly line. The opposite is true when going from forward bearing to true bearing.)
Distance of line A-G: 15921.48 ft . (departure of A-G) $\div \sin 89^{\circ} 44^{\prime} 19.01^{\prime \prime}=\mathbf{1 5 9 2 1 . 6 5} \mathbf{f t}$.
At Pt. A, the forward bearing and distance to Pt. F on the curve is:
N. 89 ${ }^{\circ}$ 44’ $19.01 "$ E., 15921.65 ft (notice this agrees with the measured forward bearing and distance of line $A-G$. The minor difference in distance is the result of rounding)

Coordinates of the proportioned point: N.10072.64, E. 25921.48


## HANDOUT

## Discussion of Convergency

Surveys may be divided into two classes: plane and geodetic. The first treats the surface of the earth as a plane; the second treats it as the surface of a spheroid. Plane surveying is suitable for small areas because the effect of curvature of the surface of the earth is not appreciable over short distances. Over longer distances, the effect of curvature becomes significant.

One way in which the effect of curvature manifests itself is in the convergence of the meridians. This convergence may be observed on a globe of the world; all meridians meet at the poles. Since they are great circles of the earth, they may be laid out on the ground as converging straight lines. Thus in northern latitudes, the distance between two meridians becomes less as one proceeds northward.

Parallels of latitude (except the equator) are not great circles of the earth. They do not converge and they may not be laid out as straight lines, either on the ground or on a map. They must be laid out as circular curves. The degree of curvature of these parallels becomes greater and greater as one proceeds north in northern latitudes; but everywhere the curvature of a parallel must be such that it crosses every meridian at right angles. The parallel of latitude are everywhere true east-west lines; and the meridians true north-south lines.

The law requires that the north-south township boundaries shall follow true meridians and that the east-west township boundaries must cross the meridians at right angles, i.e. they must follow parallels of latitude. The problem then resolves itself into one of devising a practical method of laying out these curved parallels of latitude on the ground with sufficient accuracy to satisfy the requirement of the law and the demands of good surveying practice. Three methods are commonly employed: The Tangent Method, The Secant Method and The Chord Method.

The Tangent Method: This method of laying out the latitude curve on the ground consists of : (1) orienting the instrument in the true meridian; (2) turning a 90 -degree angle right or left as the case may be, and projecting a straight line for six miles; (3) measuring half mile intervals along this straight line and at the end of each half mile measure an offset north and establishing a point on the latitude curve. The straight line, being 90 degrees to the meridian at the point of beginning, is tangent to the parallel of latitude at that point and has a true east-west direction. East of that point will have an increasingly southeasterly direction, and west of that point an increasingly southwesterly direction. Figure 14, page 154 of the manual illustrates the establishment of a true parallel east from the point of beginning. Offsets from the tangent through its six mile length are tabulated in the Standard Field Tables, Tables $12 \& 13$ page $200 \& 201$, for latitudes between 25 and 70 degrees. Intermediate offsets may be computed on the assumption that the lengths of the offsets vary as the squares of their distances from the point of beginning. Actually a series of points thus determined would define a parabola, but results thus determined are well within the expected limits of accuracy of the survey.

The Secant Method: This method is a modification of the tangent method, designed to keep all offsets relatively short. As shown in Figure 15, page 156 of the manual the secant is a straight line six miles long. It cuts the latitude curve at the first and fifth miles. It is parallel to an imaginary line drawn tangent to the latitude curve at the third mile. Its bearing is southwest or northeast depending upon the direction of projection. East of the third mile its bearing is southeast or northwest, depending upon the direction of projection. Azimuths of the secant throughout its six mile length and offsets therefrom to the latitude curve at half mile intervals are given in the Standard Field Tables, Tables 14 $\& 15$, pages 202-203, for latitudes from 25 to 70 degrees.

To lay out the latitude curve by the secant method: From a known point on the latitude curve at mile zero, measure the tabulated offset south to the zero end of the secant. With the instrument on the point thus determined and oriented in the meridian, turn off the proper angle to establish the tabulated bearing of the secant at that point. Project the secant as a straight line for six miles. As measurements are completed to each corner, measure the proper offset north or south and establish the corner. Observe that offsets are zero at the first and fifth miles.

The Chord Method: This method is a modification of the secant method. The offset from the chord are increased by the amount of the offset from the secant at mile zero. The advantages of this method are that all offsets are south (in north latitudes) and the line is started at the corner (mile zero point) without an offset.

To lay out the latitude curve by the cord method: From a known point on the latitude curve at mile zero, orient the instrument to the meridian and turn off the proper angle to establish the tabulated bearing of the secant at that point. Project the chord as a straight line for six miles. As measurements are completed to each corner, measure the proper offset south and establish the corner.

## CONVERGENCE OF THE MERIDIANS

An idea of the amount of angular convergence of the meridians may be seen from an examination of figures 14 and 15 of the manual. The values of angular convergence are shown in the Standard Field Tables, Table 11, page 199, for meridians six miles apart, in latitudes from 25 to 70 degrees. If a straight line crosses two meridians the difference between the corresponding angles formed by the line and the meridians represents the angular convergence of the meridians between the two points depends upon their difference in longitude (i.e., the departure of the line joining them) and also upon the mean latitude of the two points.

In adjusting the courses of a traverse so as to take angular convergence into account, the total angular convergence is distributed so that the amount ascribed to each course bears to the total departure of the traverse. The Standard Field Tables show the amount of linear convergence of two meridians six miles long and six miles apart, e.g., the convergence of the meridional boundaries of a township, which is to say the amount by
which the north boundary of a township is shorter than its south boundary. In using the table the mean latitude between the north and south boundaries should be employed.

In any given mean latitude the amount of linear convergence of two meridians depends upon their length and the distance between them. From this is follows that linear convergence is a function of the area, i.e., length times breadth. Thus in dealing with an irregularly shaped survey the effect of linear convergence may be regarded as a function of the area. Its amount will be in the same proportion as the area of the figure is to the tabulated values for 36 square miles (township) in the Standard Field Tables, Table 11, page 199.

## Rhumb Lines and Loxodromes

As noted above, straight lines on the earth's surface do not have a constant bearing throughout their lengths. (Exceptions are the meridians and the equator). Conversely, lines of constant bearing (excepting the meridians and equator) are curved lines. Lines of constant bearing are called loxodromes or rhumb lines. The curvature of a loxodrome may be circular, as in the case of latitudinal curves; or it may be in the form of a spiral. All lines of constant bearing,, excepting those running in the cardinal directions have the form of spirals.

Although it is possible to calculate the form of such spirals it is often regarded as impractical to do so. Frequently, sufficient accuracy may be attained by assuming that a straight line, having a mean bearing equivalent to that of the required loxodrome will serve. The mean bearing of such a line is taken to be the average of the forward bearings at its two ends, i.e., the average of its forward bearing and the 180 degree opposite of its back bearing. The foregoing assumption may lead to intolerable error, however, in the case of very long lines. A concept of the validity of the assumption may be gained from figure 15 of the manual showing the secant method of laying off the true parallel. The bearing "East" at the center is the mean between the bearings at the ends.

An acceptable method of laying out a loxodrome upon the ground by foresights and backsights with a surveyor's transit consists of calculating the angular convergence between the ends of the line and turning proportional deflection angles along the line, thus producing a series of chords which closely approximate a line of constant bearing.

## CONVERGENCY <br> AND <br> RESURVEY

## Example I:

You have been assigned to resurvey the S. boundary of Tp. $\qquad$ , R. $\qquad$ , latitude $46^{\circ} \mathrm{N}$. Your instructions call for the recovery or reestablishment of all lost corners along the line. The record bearing and distance for this line is West, 480 chs. With all intermediate corners established at 40 chain intervals along a true parallel of latitude.

## FIGURE 1

Lat. $46^{\circ} \mathrm{N}$.


In retracing the line you determined the meridian at the original corner 'A,' Figure 1 and turned west, extending the line and placing temporary corner stakes at $1 / 2$ mile intervals. When you turned west your intent was to retrace the record by turning the tangent to the record line A-C, Figure 1. The search for corners resulted in the recovery of only the original township corner 'D' at a distance of 480 chs. (Record Dist.) and 0.80 ch. North of the temp. 'B.'

According to table 13, pg. 201, Standard Field Tables, the offset from the tangent to the parallel at lat. $46^{\circ} \mathrm{N}$. is 38 lks . ( 0.38 chs.) If this corner was in accord with the record it would have been found 0.38 chs. North of your 6 mile ( 480 ch .) temp. The actual measurement from your temp to the found corner was 0.80 chs. Figure 2 shows the new configuration of the line between the two found corners.

## FIGURE 2



The curved line A-D now becomes the true line and the line A-B becomes a random line and is not tangent to the curved line A-D. Since our random line is not tangent to the true line the problem becomes one of determining the moves from the temps to the true line, keeping in mind that the true line is actually curving away from its own tangent and the random line.

The solution to the problem is simply one of solving a series of similar plane triangles and applying offsets from the tangent to the true curved line.

Figure 3 shows the construction of a tangent A-E to the true line and the forming of the plane triangle A E B.

## FIGURE 3



The offset from the tangent D-E is taken from the tables for lat. $46^{\circ}$ N. Points A and E are connected by a straight line thus forming a tangent to the true line A-D.

The offset from the tangent D-E (.38) is now subtracted from measurement B-D (.80) to obtain the distance B-E (.42) thus forming the base of triangle A E B.

The distances from the random line temps can be determined from the ratio $\mathrm{BE}: \mathrm{AB}$, (Figure 3). These results are then added to the tabulated offsets from tangent for lat. $46^{\circ}$ N. (Standard Field Tables, Table 13)

$$
\frac{42}{480}=.000875 \text { (Constant Multiplier) }
$$

$\left.\begin{array}{|l|l|c|c|c|c|c|c|c|c|}\hline \text { Temp. } & \text { Dist. } & \text { Constant } & & \text { Proportion } & \begin{array}{l}\text { O.S. From Tan. } \\ \text { Table 13, 46 }\end{array} \\ \hline\end{array} \begin{array}{l}\text { Move from } \\ \text { Temp. }\end{array}\right]$

## Example II:

Your instructions call for the recovery or reestablishment of all corners on the west four miles of the N. Bdy. Of Tp. $\qquad$ , R. $\qquad$ , in latitude $51^{\circ} \mathrm{N}$. The record is West, 320 chs. with all intermediate corners established at 40 chain intervals along a true parallel of latitude.

Figure 4


In retracing the line you determined the meridian at the original corner 'A,' Fig. 4 and turned West, extending the line and placing temporary corner stakes at $1 / 2$ mile intervals. When you turned West your intention was to retrace the record by running the tangent to the record line A-C. The search for corners resulted in the recovery of only the original township corner 'D' at a distance of 320 chs. (Record Dist.) and 0.15 chs. South of the temp. ‘B.'

According to table 13, page 201, Standard Field Tables the offset from the tangent to the parallel for 4 miles, in lat. $51^{\circ}$ is 20 lks . ( 0.20 chs.) If this corner was in accord with the record it would have been found 0.20 chs. North of temp. B. The actual measurement from the temp. to the corner was 0.15 chs. South. Figure 5 shows the new configuration of the line between the two found corners.

Figure 5


The curved line A-D now becomes the true line and the line A-B becomes a random line and is apparently not tangent to curved line A-D. The problem of determining the moves from the random line temps to the true line is basically the same as that in Example I.

Figure 6 shows the construction of the tangent A-E and the formation of triangle A E B.

Figure 6


As in Example I, we can now compute the values of the lines from the temps to the tangent and by subtracting the offsets from the tangent for $51^{\circ}$ we have the moves from the random line temps to the true line; in this case all moves are south.

Then:
$\frac{\mathrm{BE}}{\mathrm{AE}}=\frac{.35}{320}=.001094($ Constant Multiplier $)$

| Temp. | Dist. | Constant |  | Proportion | O.S. From Tan. <br> Table 13, 51 | Move From <br> Temp. |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 40 | x | .001094 | $=$ | .044 | - | .00 | $=$ |
| 2 | 80 |  | $"$ |  | .087 |  | .04 |  |
| 3 | 120 |  | $"$ |  | .131 |  | .03 |  |
| 4 | 160 |  | $"$ |  | .175 |  | .077 |  |
| 5 | 200 |  | $"$ |  | .219 |  | .08 | .101 |
| 6 | 240 |  | $"$ |  | .262 |  | .11 | .125 |
| 7 | 280 |  | $"$ |  | .306 |  | .15 | .159 |
| B | 320 |  | $"$ |  | $* .350$ |  | .20 | .156 |

In the foregoing examples we have retraced the record line by running the tangent to the true parallel of latitude. After locating the controlling corner the line that was run as a tangent to record the true line loses its value as a tangent to the true line because the true line has assumed a position other than that of the record. Therefore the line run as a tangent becomes a random line. From this point it can be seen that the random line does not need to be tangent to the record line. It can be any line run in the general direction of the controlling corner. It can be the chord or secant of the record - or either of these. In any case the calculations can be performed in the same manner as shown in the examples.

The method we have discussed is a close approximation. The results obtained are, for all practical purposes, well within the required accuracy for land surveys as long as the survey is not extended beyond the limits of the values tabulated in the Standard Field Tables.

## OFFSET FROM TANGENT

OFFSET IN LINKS =

$$
=1.01 \times \text { TAN } \mathrm{F} \times \mathrm{MILES}^{2}
$$

(SIN BRG.)

$$
=0.000158 \times \text { TAN F x CHAINS }{ }^{2} \quad(\text { SIN BRG. })
$$

F = MEAN LATITUDE

## EXAMPLES:

LINE $=$ N. $80^{\circ} 00^{\prime}$ E., 640 CHS. ( 8 MILES)

$$
\mathrm{F}=41^{\circ} \quad 30^{\prime} \mathrm{N} .
$$

OFFSET $=1.01 \times .88473 \times(8)^{2} \times .98481=56.32$ LKS.
OR
OFFSET $=.000158 \times .88473 \times(640)^{2} \times .98481=56.39$ LKS.

## OFFSETS FOR TANGENTS FACTORS FOR OFFSETS IN LINKS

| LAT. | "C" Factor | LAT. | "C" Factor | LAT. | "C" Factor |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $30^{\circ} 00^{\prime}$ | 0.90971 | $45^{\circ} 00^{\prime}$ | 1.57433 | $60^{\circ} 00$ | 2.72451 |
| $\begin{array}{ll}30 & 30\end{array}$ | 0.92811 | $45 \quad 30$ | 1.60200 | $60 \quad 30$ | 2.78018 |
| 3100 | 0.94670 | $46 \quad 00$ | 1.63013 | 6100 | 2.83743 |
| 3130 | 0.96549 | $46 \quad 30$ | 1.65885 | $61 \quad 30$ | 2.89687 |
| 3200 | 0.98448 | $47 \quad 00$ | 1.68806 | $62 \quad 00$ | 2.95807 |
| $\begin{array}{lll}32 & 30\end{array}$ | 1.00368 | $47 \quad 30$ | 1.71783 | $62 \quad 30$ | 3.02131 |
| 33 00 <br> 33  | 1.02309 | $48 \quad 00$ | 1.74816 | $63 \quad 00$ | 3.08671 |
| 33 30 <br> 34 00 | 1.04272 | $48 \quad 30$ | 1.77909 | $63 \quad 30$ | 3.15439 |
| 34 00 | 1.06257 | $49 \quad 00$ | 1.81063 | $64 \quad 00$ | 3.22447 |
| 34 30 <br> 35 00 | 1.08266 | $49 \quad 30$ | 1.84281 | $64 \quad 30$ | 3.29713 |
| 35 00 <br> 35  | 1.10300 | $50 \quad 00$ | 1.87566 | $65 \quad 00$ | 3.37247 |
| 35 30 <br> 36 00 | 1.12358 | $50 \quad 30$ | 1.90919 | $65 \quad 30$ | 3.45071 |
| $36 \quad 00$ | 1.14442 | 5100 | 1.94345 | 6600 | 3.53199 |
| 36 30 <br> 37 00 | 1.16552 | 5130 | 1.97844 | $66 \quad 30$ | 3.61652 |
| $37 \quad 00$ | 1.18690 | $52 \quad 00$ | 2.01422 | $67 \quad 00$ | 3.70452 |
| 37 30 <br> 38 00 | 1.20856 | $52 \quad 30$ | 2.05080 | $67 \quad 30$ | 3.79619 |
| 38 00 <br> 38  | 1.23050 | $\begin{array}{lll}53 & 00\end{array}$ | 2.08824 | $68 \quad 00$ | 3.89185 |
| $\begin{array}{lll}38 & 30\end{array}$ | 1.25276 | $53 \quad 30$ | 2.12652 | $68 \quad 30$ | 3.99171 |
| 39 00 <br> 39 30 | 1.27532 | $54 \quad 00$ | 2.16574 | $69 \quad 00$ | 4.09610 |
| $\begin{array}{ll}39 & 30\end{array}$ | 1.29820 | $54 \quad 30$ | 2.20591 | $69 \quad 30$ | 4.20535 |
| $40 \quad 00$ | 1.32141 | $\begin{array}{ll}55 & 00\end{array}$ | 2.24707 | $70 \quad 00$ | 4.31981 |
| $\begin{array}{lll}40 & 30\end{array}$ | 1.34496 | $55 \quad 30$ | 2.28927 | $70 \quad 30$ | 4.43992 |
| 4100 | 1.36887 | $56 \quad 00$ | 2.33256 | 7100 | 4.56616 |
| 41 30 <br> 4  | 1.39314 | $56 \quad 30$ | 2.37698 | $\begin{array}{ll}71 & 30\end{array}$ | 4.69969 |
| 4200 | 1.41778 | $57 \quad 00$ | 2.42258 | $72 \quad 00$ | 4.83864 |
| $42 \quad 30$ | 1.44282 | $57 \quad 30$ | 2.46943 | $72 \quad 30$ | 4.98620 |
| 4300 | 1.46826 | $58 \quad 00$ | 2.51758 | $\begin{array}{ll}73 & 00\end{array}$ | 5.14217 |
| $43 \quad 30$ | 1.49411 | $58 \quad 30$ | 2.56709 | $73 \quad 30$ | 5.30727 |
| $44 \quad 00$ | 1.52040 | 5900 | 2.61804 | $74 \quad 00$ | 5.48246 |
| 44 30 <br> 45  | 1.54713 | 5930 | 2.67048 | $74 \quad 30$ | 5.66859 |
| 4500 | 1.57433 | $60 \quad 00$ | 2.72451 | $75 \quad 00$ | 5.86687 |

$C=$ " C " factor from above table. D=Distance in chains, East or West
$M=$ Move, in links, from the Tangent to the Parallel
$\mathrm{M}=(\mathrm{D} \backslash 100)^{2} \mathrm{C}$

## CONVERGENCY OF MERIDIANS

NOTATION:
C = CONVERGENCY IN SECONDS
F = MEAN LATITUDE
$\mathrm{C}=.6501 \times$ TAN F x CHAINS x SINE BRG.
C $=52.008 \times$ TAN F x MILES x SINE BRG.

## EXAMPLE:

LINE=N. $80^{\circ} 00^{\prime}$ E., 640 CHS. OR 8 MILES

$$
\mathrm{F}=41^{\circ} 31^{\prime} \mathrm{N} .
$$

$\mathrm{C}=.6501 \times .88473 \times 640 \times .98481=362.51 "$
OR
$C=52.008 \times .88473 \times 8 \times .98481=362.51 "$


> Mean Latitude
> 41 degrees and 30 min.

Example: I have run a straight line ( AB ) on a beginning bearing of N. $68^{\circ} 30^{\prime} 00^{\prime \prime} \mathrm{E}$., for a distance of 200 chs. I have computed the angular convergency to be $0^{\circ} 01^{\prime} 47^{\prime \prime}$. What is the bearing of the line at B ?

Rule: If the latitude of the line is increasing with its direction, add the correction for convergency.
If the latitude of the line is decreasing with its direction, subtract the correction for convergency.

Therefore : The beginning bearing is $\mathrm{N} .68^{\circ} 30^{\prime} 00^{\prime \prime} \mathrm{E}$. Since the latitude of the course is increasing, I add the convergency: $68^{\circ} 01^{\prime} 00^{\prime \prime}+0^{\circ} 01^{\prime} 47^{\prime \prime}$ and the bearing of the line at point B becomes N. $68^{\circ} 31^{\prime} 47^{\prime \prime} \mathrm{E}$.

## LINEAR CONVERGENCY

The linear Convergency of a township or of meridians is an amount expressed in linear measurement; e.g., miles, chains, links; of the effect due to the meridians converging at the poles.

Bluntly: We are doing plane surveying on a round surface and must make adjustments accordingly.

## LINEAR CONVERGENCY OF MERIDIANS

Linear convergency of survey figures may be computed as follows:
(A) Using BLM STANDARD FIELD TABLES in Table 11 with argument as the approximate mean latitude (see example 1).
(B) By the formula

$$
\mathrm{dm} ?=\underline{\mathrm{m}} ? \mathrm{mF}
$$ $\tan \mathrm{fv} \overline{\left(1-\mathrm{e}^{2} \operatorname{Sin}^{2} F\right)}$

(see example 2)
(C) For fractional parts of a township. (see example 3)

The 1947 Manual, Section 129 states:
"Simple interpolatioin may be made for any intermediate latitude, and the amount of the convergency for a fractional township or other figure may be
taken in proportion to the tabulated convergency as the fractional area is to 36 square miles."
(D) For irregular figures. (see example 4)

The 1947 Manuel, Section 129 also states:
"The correction for convergency in any closed figure is proportional to the area and may be computed from an equivalent rectangular area."
(E) For a single course. (see example 5)

## EXAMPLE I

Linear convergency of a township ( 6 miles long by 6 miles wide) at mean latitude $48^{\circ}$ $00^{\prime} \mathrm{N}$., can be looked up directly in the STANDARD FIELD TABLES in Table 11.

For this example the linear amount of convergency is 80.6 links.
With the same data at mean latitude $26^{\circ} 30^{\prime} \mathrm{N}$., a straight line interpolation is made between mean latitudes $26^{\circ} 00^{\prime}$ and $27^{\circ} 00^{\prime}$ to obtain 36.2 links as the amount of convergency in the township.

With the same data at mean latitude $42^{\circ} 00^{\prime} \mathrm{N}$., the convergency would be 65.4 links.
Note: Compare with example 2.

## EXAMPLE 2

The linear convergency of meridians of a township or line may be found by the formula in the STANDARD FIELD TABLES (page 224):

| $\mathrm{dm} ?$ | $:$ | $\frac{\mathrm{m} ? \mathrm{mF}}{\mathrm{a}}$ |
| :--- | :--- | :--- |
| $\mathrm{dm} ?$ | $:$ | Linear amount of convergency |
| $\mathrm{m} ?$ | $:$ | $\tan \mathrm{fv} \overline{\left(1-\mathrm{e}^{2} \operatorname{Sin}^{2} \mathrm{~F}\right)}$ |
| mF | $:$ | Measurement along the parallel |
| a | $:$ | Equitorial radius of the earth 317064 chains |
| e | $:$ | Factor of eccentricity loge e $8.9152515-10$ |
| $\mathrm{e}^{2}$ | $:$ | e in natural numbers squared 0.006768658 |

F : Approximate mean latitude
e was given in log form because previous calculators were not made to handle the squaring and multiplication under the radical with proper precision.

Given : mean latitude of $42^{\circ} 00^{\prime} \mathrm{N}$.

$$
\left.\begin{array}{ll}
\mathrm{dm} ?= & \frac{\mathrm{m} ? \mathrm{mF}}{\mathrm{a}} \\
\mathrm{dm} ?= & \frac{(480 \mathrm{chs})(480 \mathrm{chs})}{317064 \mathrm{chs}} \\
\mathrm{tan} \mathrm{fv}\left(1-\mathrm{e}^{2}\right. & \left.\operatorname{Sin}^{2} \mathrm{~F}\right) \\
\mathrm{tm} ?= & (.72667 \mathrm{chs})(.90040)\left(\overline{\mathrm{v}} \overline{\left.\mathrm{v} \overline{\left(1-\mathrm{e}^{2} \mathrm{e}^{2} \sin ^{2} \operatorname{Sin}^{2} 42^{\circ}\right.}\right)}\right. \\
\mathrm{dm} ?= & (.72667 \mathrm{chs})(.90040)(\mathrm{v} \overline{1-(0.006768658)(0.4477357686})
\end{array}\right)
$$

## EXAMPLE 3

This method is very appropriate for finding the linear convergency for any portion of a township: e.g., one section, two sections, etc., or any closed figure at the same approximate mean latitude.

METHOD: $\frac{\text { conv. (from Table 11) }}{36 \text { square miles }}=\frac{\text { Conv. }}{\text { "Any Area" }}$
(1) Given: mean latitude $49^{\circ} 00^{\prime} \mathrm{N}$.

Find: Conv. For 2 sections

$$
\begin{aligned}
& \frac{83.5 \text { links }}{36 \text { Square miles }}=\frac{\text { Conv. }}{2 \text { square miles }} \\
& \text { Conv. }=04.6 \text { links }
\end{aligned}
$$

(2) Given: mean latitude $34^{\circ} 30^{\prime} \mathrm{N}$.

Find: Conv. For 8.4 square miles

$$
\begin{aligned}
& \frac{50.0 \text { links }}{36 \text { square miles }}=\frac{\text { Conv. }}{8.4 \text { square miles }} \\
& \text { Conv. }=11.7 \text { links }
\end{aligned}
$$

## EXAMPLE 4

The area of any closed figure can be described by an equivalent rectangular area. e.g., an area of 9.7 square miles can be reduced to an equivalent rectangular area of a rectangle 4.85 miles by 2 miles or 2.425 miles by 4 miles, etc.

Formula: Conv. $=.0202 \mathrm{~d} 1 \tan \mathrm{~F}$
(d) (1) is length times width in miles which yields area
(1) Given: mean latitude $32^{\circ} 00^{\prime} \mathrm{N}$

Area of figure $=22.8$ sq. miles
Find: convergency in chains

$$
\begin{aligned}
& \text { Conv. }=(.0202)(2 \text { miles })(11.4 \text { miles })\left(\tan 32^{\circ}\right) \\
& \text { Conv. }=.287 \text { chains }
\end{aligned}
$$

Note: This formula yields answer in chains. May be used for townships or any portion of a township.
(2) Given: mean latitude $34^{\circ} 30^{\prime} \mathrm{N}$.

Find: Conv. For 8.4 square miles

$$
\begin{aligned}
& \text { Conv. }=(.0202)(2 \text { miles })(4.2 \text { miles })\left(\tan 34^{\circ} 30^{\prime}\right) \\
& \text { Conv. }=.117 \text { chains }
\end{aligned}
$$

Note: Compare answer to example 3 part 2

## RESURVEY METHODS <br> CURVATURE (SECONDARY METHOD)

An alternate method of computing the curvature and incorporating its effect into corner moves along a standard parallel or township line is by the mean bearing method.

In this method the amount of curvature for the latitude of the line is taken from the standard field tables and the proper amount applied to the bearing of the portion of the random line concerned. The latitudes and departures of the traverse are computed and accumulated and the mean bearing of the true line is determined. The true line proportional latitudes and departures are then computed and accumulated. The difference between the true line accumulated data and the random accumulated data for each increment of the survey line is the move. The move then includes the adjustment for curvature.

## Course 4: Restoration of Lost Corners Study Guide

## COURSE DESCRIPTION:

## COURSE

 OBJECTIVES:
## COURSE INSTRUCTOR(S):

VIDEO LECTURE TITLE:

This course consists of four videos, some reading, and three exercises, on the "Restoration of Lost Corners". The legal, mathematical, and practical applications of the methods of proportioning, as found in the Manual of Surveying Instructions, are presented. Students will be able to address what corners control in most situations, how to proportion properly, what legal principles are involved when proportioning, and how to deal with the latitudinal curve. A lengthy discussion of convergence and curvature in the PLSS is also included.

Upon completion of this course, students will be able to:

- Define the three corner conditions listed in the Manual of Surveying Instructions
- Describe, identify applicability, and compute proportions using all methods
- Demonstrate an understanding of curvature in the PLSS

Dennis Mouland, Bureau of Land Management
Ron Scherler, Bureau of Land Management

Restoration of Lost Corners - Part 3 (55 minutes)

| ICON LEGEND |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | EXERCISE | $\square$ <br> DIAGRAM |  | PROBLEM |  |  |  |

## Restoration of Lost Corners - Part 3

## Introduction

Welcome back, this is Video Lecture 3 of Restoration of Lost Corners here in the CFedS courses. Ron just finished talking to you about curvature and convergency. I know that this is new to most of us that have not been born and raised in the BLM.

I learned quite a bit in that session as well because I had learned other ways. They come out the same but I had learned other ways to do these things. So very educational. I hope that you found it useful. And of course the advantage of being on DVD is that you can play it back over and over again. Well now we are going to completely shift gears. All of that in the previous discussion was all about single proportioning and we ended with single proportions on lines that are curved.

Well now it is time to move on to double proportions. And so let's take a look at some things there. Double proportions are described in Section 7-8 of the Manual and I will read that to you in a minute.

It applies to section corners and township corners but only where corners were truly established in the four way establishment. That is a problem with completion surveys that you will see and it requires the use of cardinal equivalents and true bearing.

## Double Proportions

- Described in 7-8 of Manual
- Apply to section corners and township corners
- Only corners with true 4-way establishment
- Requires the use of "cardinal equivalents" and True Bearing

Now true bearing of course we keep talking about it and the Manual expects that you will always be on true bearing and we expect you to always be on true bearing. Let's go to 7-8, I realize that you have read this as part of your assignment, but I want to focus on just a few sentences here. 7-8 the term double proportion of measurement is a applied to a new measurement made between four known corners to each on intersecting meridianal latitudinal lines for the purpose of relating the intersection to both.

Now frankly that is a terribly written sentence as to what is really going on here because it makes it sound like this is an intersection of lines in the sense of a bearing-bearing intersection which it is not.

## Restoration of Lost Corners - Part 3

Now a lot of surveyors believe that a double proportion is a bearing bearing intersection. That is not true. And I even know some state tests that have been conducted for surveyors where that is that is the answer they are looking for that is a bearing bearing intersection. That is totally inappropriate.

In effect, it says here by double proportionate measurement the record directions are disregarded. The directions are going to be disregarded. Now how is that and why is that? That is an interesting question. Now if go down in that second paragraph down about two-thirds of the way you will see a sentence that is in italics.

And it is in italics for a reason and you might want to highlight it yellow in your Manual because in my opinion it is the most misunderstood sentence in the entire book. Lengths of proportioned lines are comparable only when reduced to their cardinal equivalents. They are comparable only when they are reduced to their cardinal equivalents. Now what in the world does that mean?

Well let's take a look at the slide. What is a cardinal equivalent? Lengths of proportioned lines are not comparable until reduced to their cardinal equivalents.

Cardinal is one of the four points of the compass and what so we are doing, it is just the same computation as when you reduce for a slope distance except we are in a horizontal plain here.

So what we are doing in that drawing at the bottom of the slide is that we are looking down on a situation and maybe the actual inversed horizontal distance is on you know say looking at that guessing maybe north 88 degrees west, but the cardinal equivalent is not the distance along that line between the two corners, the

## Restoration of Lost Corners - Part 3

cardinal equivalent is what is the difference in eastings. In other words what is the distance between those but only in the cardinal direction of that line. So that would be east.

So you know if this was using the slope distance analogy, if that was north 88 west there, then you would take the co sine of two degrees times whatever your inverse distance is and that would give you the cardinal equivalent. That is what a cardinal equivalent is and it is required in the Public Land System for a number of things and in particular for double proportions.

So now let's think about this for a minute because the cardinal equivalent suddenly brings up some issues that you maybe you've already thought about but you know most surveyors and even a lot of software that are supposedly automatically double proportions that sort of thing.

It doesn't use the correct distances, it does not use the cardinal equivalents. It uses the actual inverse distance, which is not what the Manual says to do. Now I recognize one of the problems in the Manual, see figures 7-1 and 7-2, you've got those diagrams there.

That middle diagram that looks like a gun telescope or cross hairs you know that is very poorly drawn as far as how this is supposed to really relate. Because that is how people think that this is some kind of a bearing-bearing intersection or some other solution.

What you have to understand is that line AB and line CD in that drawing is cardinal. They are already reduced to their cardinal equivalency. So you see most surveyors are using the wrong distances in a double proportion, they are using the actual distance from point to point and that is not what the book says and in fact it specifically said that they are only comparable when reduced to their cardinal equivalence.

Really when you think about it, there is some logic in that. If you've got a line running at this bearing and then another one due east and how far is it from one end of the line to the other? Well do you add this length and this length? But you see that is not really how far they are and we are not even interested in the

## Restoration of Lost Corners - Part 3

distance inverse between the two ends, we are interested in the cardinal only.

That way the corners to the east and west only control the east west position their north south relationship the east west corners, their north south relationship to each other is not a factor.

That is why the Manual said that in effect the record directions or bearings are disregarded. We remove them so we bring everything to apples. See otherwise when you do a double proportion you've got in the example we will have here in a few minutes, you have apples, oranges, you know, watermelons, and lemons. You've got things at all different kinds of bearings and adding and subtracting and dividing those is not a correct way to do a proportion because it starts to weight one line over the other where it shouldn't.

And so to do a cardinal equivalent, we reduce everything to cardinal equivalents so that we can add and subtract and multiply and divide those numbers and everything is on the common ground, that being cardinal directions.

And then so obviously, flipping those corners to the north and south, they will control the north south position of the corner but the east west relationship for the north and south corners will not affect that. That's what's going on here.

Now cardinal equivalents then. A very important question we want to ask then is, how does your basis of bearings affect the cardinal equivalent?

You see this is why you've got to be on astronomic which the Manual assumes you are all the time because if you are on any other basis of bearing, you will change the answer. You will change the location of your lost corner.

## Cardinal Equivalents

- How does your "basis of bearings" affect the cardinal equivalent?
- The Manual assumes you are on an astronomic basis at all times
- If you are any other basis, you will change the answer and location of the lost corner.
- Do you see how it "disregarded the bearings"?



## Restoration of Lost Corners - Part 3

Let's just think about that for a minute. If you have the example there that I used you know the north 88 west, whatever the co sine is 2 degrees times half a mile, you know you can do that yourself. And then say you are on different basis of bearings. You are on one where you think the inverse distance actually is east. So now there is no cardinal reduction it's just the half mile.

Well compare those two and you will see that those are, you know, I don't know I don't have my calculator here in front of me but you know what I'm saying? That you did that you know it's probably going to make a foot or two difference in what number you are going to proportion against, of course you are going to do that four times. You're going to make that error four times because your lines are at different bearings.

So there is no way to predict how far you will be off. I know that in the sample problem we'll do here in a few minutes if you were I think its two degrees off for your basis of bearings, we just did a what if.

It will actually affect, you'll move the corner a little over three feet. So you know this is worth thinking about and realizing that your basis of bearings is absolutely critical at all times and not that I want to say well there are a few times that you can ignore it, I mean obviously with a single proportion on a straight line, you're going to ignore it?

But your whole project as a CFedS and frankly as any kind of surveyor doing should be on true bearing and so what that does is allows you to do the correct cardinal reductions or cardinal equivalents with these projects. So that is how we disregarded the bearings.

## Restoration of Lost Corners - Part 3

So now let's run through a double proportion process, just list the steps and then I'll show you how to do that. The first step is that we are going to reduce all of the record distances to cardinal equivalents. That is what ce stands for there. And then you are going to reduce all of your measured distances to cardinal equivalents.

Because you see to compare these distances in math, they have to all be cardinal equivalents. We will then compute the proportion north and south and the east west and this gives us two different, you know the north south will give you a northing and the east west will give you an easting and then where those intersect or in other words cardinal moves, you're going to have an answer.

Now what you have to, that will give you the position of your corner, but what you have to remember is that if you reduce the record to cardinal and the measured to cardinal then the answer you get is cardinal. So you are not going to take that cardinal distance that you get and run up the line at some bearing, you are going to go due north south or due east west with that number. So we can establish the lost point.

I'll show you how to do that quite easily especially with coordinates and then from that coordinate you will have to inverse back out to your controlling corners to get what your bearings actually are. So that is kind of the step by step process of a double proportion. I realize that this is new to many of you that is because this is just one of those things that just has never been taught or properly taught even in many of our colleges and universities. So let's do a sample problem here.

## Restoration of Lost Corners - Part 3

This screen shows you the record data of a double proportion and you can see there is some what we call some heavy bearings in there this is obviously a section corner that had been found by a previous retracement.

It would be very rare that you would have the original survey show this at this kind of heavy bearings. That would all have been usually in most parts of the country within the 21 Minute Rule. But you can see that three of the four lines coming into this lost section corner are what we call heavy, you know they are outside of the limits but so how did this happen?

It is because of a retracement. So we have to take every one of those numbers based on their corresponding bearing and reduce them to cardinal equivalents.


Now the next slide is the measured data. And what I have done is just given you coordinates and just assume that you have either traversed through those or GPS'd it whatever this is just simple coordinates in chains, all right? But what do we do with these? So let's do the north south computation first.


## Restoration of Lost Corners - Part 3

As you can on this slide, we are going to do two separate single proportions. Now that we have looked at our record and measured data and discussed the process here, now we are going to do two separate single proportions. Remember that we are going to do them as cardinal equivalents. Everything in here will be cardinal.

Let us look at this north-south computation first. 120.875 in the measured. Where did I come up with that? That is measured data. Let us go back to the measured data slide. Remember, that if you are on the correct basis of bearings which you must be and you are using coordinates - these are in chains. They can be in anything. Where these corners are based on the proper basis of bearings that the difference of the northing there, and the northing there is a cardinal equivalent. Because the northing difference only will be right up at due north line.

So, we don't have to do any computations on the measured data if you are using coordinates to come up with a computation. That is where the 120.875 comes from for the measured because we can subtract between our coordinates to get that. Now the record, we are going to have to do by hand if you will. Where did we come up with these numbers here? Let us take a look at that.

On the record data, the north/south we are doing. The first line the line to the north, it is only one minute from cardinal, so you could take the co sine from one minute times 40.09 and it will not come out to anything other than 40.09 unless you carry it way out. My point is there is no cardinal equivalent there. The cardinal equivalent for that line is 40.09 . But, on the southerly portion the mile that we are going to do here is one degree twenty minutes from cardinal.

In order to get the cardinal equivalent for the record on this line is we are going to have to take the co sine of one degree twenty minutes and multiply that times 80.66 and that is going to give us these two numbers here. Forty point zero nine (40.09) because there was no cardinal equivalent and 80.638. The computation is in this case because the measured is on top.

The measured over the record for the whole mile and then X is to I am going to solve X for the mile. Actually, it is a mile and half over here. A mile and a half and I am going to solve for the


## Restoration of Lost Corners - Part 3

southerly mile. Now when you do this computation, you come up with 80.736. Let us remember that is all of the numbers we used in here were cardinal and then that answer is cardinal. To be able to - this makes it simple when working with coordinates because you can take that number and add it to the northering of the southerly section corner. That is because I solved for the southerly portion, so let's see what that looks like.

I would take the 100 that is down here and I am going to add to it the 80.736 that I just computed. That is a northing only - a cardinal number. What that is giving me is a coordinate for somewhere up here wherever that section corners is going to be. It is lost so we are computing it now. It is going to give me that coordinate. 100 and 80.736 is the northing of the lost section corner. Similarly, on the east/west computation, let us run through that.

I have got in the measured 80.300, so where did that come from. The exact same thing we did last time. I am going to take easting of my easterly corner over here 142.055 and I am going to subtract from it 61.755 because the difference of those easting's is a cardinal equivalent.

That is where the 80.30 comes from for the measured. Then for the record 40.095 and 40.976 where do those come from, well we have to go back to the record data. Now these are heavy bearings here on both sides. I am going to have to take this bearing South $88^{\circ} 25$ West, 40.11 chains.

Now I am going to have to compute the difference from cardinal, which is 1 degree 35 minutes. So I am going to take the co sine of 1 degree 35 minutes multiply it times 40.11 and that is going to give me 40.095 then, I am going to do the same on this other side. 87.40 northwest, that's what 2 degrees 20 minutes, so I am going to take the co sine of 2 degrees 20 minutes and I am going to multiply that by 41.01 and that is going to give me 40.976 and so that is where these numbers came from here.

All right we are doing the east-west now, so the computation is 80.30 in the measured is to sum of those two we just computed 40.095 and 40.976 as X is to 40.095 and I am going to solve for the westerly half mile and I am doing that just because it is going

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to make it real easy for me to compute the answer. So when we solve that we get 39.714. What are we going to do with that. Once again Remember that is a cardinal number.

I can go back to my measured data where I have my coordinates that are in the proper basis of bearings and I can take the 61.755 that is there and I can add to it the 39.714 which is the number I just computed for the easting difference (cardinal equivalent). What does that give me? It gives me 101.469 as you can see down here. That is the answer.

The way I have shown you to do this is proper and what it does for you is produce independently two single proportions but it produces the northing and the easting of the lost corner. The last thing you will do is inverse back to the controlling corners.

I will run through that again here in a minute or two and review for you just how that process works and make sure we understand that but let's make sure we recognize that bit maybe oversimplify in it if I say oh yeah double proportion that is just two single proportions. No. Double proportion you heard this the last hour or so it has to be on the correct basis of bearing true meridian. Then we have to use cardinal equivalents. The cardinal equivalents must be in the record and in the measured, so you have to adjust those now.

A lot of the GLO records don't need to be adjusted because they were already in cardinal or close enough. That's fine. This problem that we just worked the record was one of the lines was close to cardinal the other three were heavy bearings as we call them, so they require cardinal equivalent. It always does and so the measured you do that and that is really easy as you just saw because when you have coordinates because you can derive your cardinal equivalents without having to take the co sine of a bearing, or two bearings, adding it or whatever.

You can simply take your coordinate differences to get your cardinal equivalents in the measured. Then we take that information and apply it to the two single proportions now using cardinal values only and we come up with a cardinal value, which makes it really easy if you are using coordinates, which we all are anymore, you can add or subtract depending upon which side of

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the line you are going to solve for you can come up with that answer.

So you know I have heard people say well I have checked somebody's double proportion and they double proportioned and I missed them by two or three feet. Well hey it could be you. You could be wrong because you did not use cardinal equivalents or on the correct basis of bearings, so I would be very cautious of that and make sure that I am fully aware of how I did it versus how they did it.

Whether I want to accept or reject somebody else's corner because it can make a difference. I encourage you to experiment with it and see how much of a difference it makes.

I am going to review that double proportion process for a moment for you. Now you will understand what I was saying. The first step was to reduce the record distances to cardinal equivalents, then reduce the measured distances to cardinal equivalents, compute your proportions just like I did, north south separate from the east west obviously.

Remember that the answer you get out of those computations is cardinal. You are able to add or subtract depending on the proper direction that answer to one of your coordinates in your measured situation that allows you to establish the lost point you will come up with the northing and the easting coordinates of the lost corner and then you want to inverse from there from that corner out to your controlling corners because you don't know your bearings and distances to it.

What you have done is a double proportion on cardinal. So once you have determined that point, you are going to have to continue to inverse back out to get the final answers. To get your actual bearings and distances going into that section corner.

I encourage you to watch this again, if you have any trouble with that because it is a little different but the really important things are don't forget to reduce to cardinal equivalent, don't forget to be on your astronomic basis of bearing, your true bearing, and then apply it as we have shown you here and you'll do just fine.

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You might say well it doesn't make that much difference the record that I am using is all within 21 minutes, are cardinal, my measured stuff is pretty close too. Well you know I don't know where the cut off is that is kind of a dynamic relationship there. I would just say do it right every time and you never have to worry about well I wonder if I was close enough or I'll guess that I am close enough. Well don't worry that, just do it.

What did it take? I went pretty slow there, but you can compute these with a calculator in two or three minutes and or you could write a software routine that does it by the book because that is what we just did was by the book. And come up with a solution.

But the bottom line here is double proportions you know the example that I was going to give you bottom line is if you go out there on the ground, and let's say somebody else, you can't find any original evidence of the section corner and no one else could either and so they proportioned it, they double proportioned it well you tie them in and you miss them by two feet now you're going to decide am I going to accept them or not.

Well I will tell you what, before I make that decision I'd want to make sure that I did this on the proper method because if I am on the wrong basis of bearings that is why I am missing them two feet. And they did it properly and they were on the correct of basis of bearing. So do you see what I am saying, to evaluate to properly and legally, and if I can even say fairly evaluate other people's positions, you really need to be careful of what your position is and how it was determined before you compare it to theirs and say well they're two feet off. Well the reason they're two feet off, or five feet off or twelve feet off.

The reason they're off could very likely be because they were paying close attention to the rules and you weren't or vice versa. So let's pay close attention to those kinds of things.


EXERCISE Before moving on to the next topic, complete the "Double Proportion Exercise" which can be found in the Exercise section at the end of this study guide.

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We are still under double proportions but these point control methods again are for this situation, you use them where all lines run by the GLO in other words you know section corners normally have four lines coming to it.

But may one or two of those weren't run by it would have been double proportioned if they had run all the lines or if possibly you have identified a blunder in one of the lines then you are going to adapt the double proportion to a different method. These are all going to require cardinal moves so once again your basis of bearings is important and it is mandatory that you pay attention to it, in other words true bearing.

And all three of the methods we are going to look at require the running of the record measurement, so a record measurement, now what does that mean? We need to think about that because that is called, it allows you to run, it tells you to run record but the Manual has some interesting things to say about an index and what they mean by the record. So let's ask this question first before we go into these.

## Point Control Methods

(Double Proportions)

- Used where all lines were not run by the GLO, but would have been double proportioned if they had or you have identified a blunder
- Require cardinal moves; therefore our basis of bearings is important.
- All three methods require the running of a "record" measurement", which includes the application of an index.

So, what is an "index"?


HANDOUT Take a few minutes to read Jerry Wahl's The Double Proportioning Made Complex paper which can be found in the Handouts section at the end of this study guide.

## Restoration of Lost Corners - Part 3

## Indexing

Let's ask this question what is an index and how does this work and how can I apply that? Indexing record is what they actually did and not just what the plat or notes say.

Now that is an interesting statement because we are all used to the record being what did the plat say and what did the notes say and that is it. But let me read you something because the Manual is interesting here. It talks about this in two different places in Chapter 7-14, 7-15 and Chapters 7-56 thru 7-58.

Let me read you the one from 7-14, this is the one, two, three, fourth paragraph in 7-14. An index correction for average error in measurement, which means it could be bearings and distances, and/or distances, if applicable should be made in applying these two rules and then they refer to section 7-56.

But then a real interesting sentence here, what is intended by record distance is the measure established in the original survey. Now that is a short sentence, but what that said is if they actually went a different distance than what the plat says and you are able to proof that, then may you ought to do that instead.

Now this is because indexing is a situation where you are going to adjust our record bearing that you don't have our measured, or excuse me record bearing or distance, that you don't have a measured bearing or distance directly on. How does that work? What is that?

Now let's understand. I'll show you here. Indexing 7-14, 7-15 ad $7-56$ thru 7-58 it can be applied to bearing and/or distance really, but it requires a substantial retracement that reveals a consistent, and I'm going, it is in italics, you may want to underline it, consistent pattern of error in the record measurements. Now here is what we've got.

Let's take a look at this slide. I am doing this a little out of order here so it threw everybody for a loop.

Take a look at this plat. Here is a township, I think in Oregon, where these sections out here were never surveyed. They are

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unsurveyed. And so what we have here is these sections are 16 here and so they came up and they set their section corner here and they ran over here and so we have returns on both of these lines but there are no returns on the other two lines, so what we have here is a situation which would be a two-point control problem which we'll look at here in a minute.

But I am showing you where indexing comes into play. You see for me to compare, you know to run this line up my record is simply 80 chains and if that is all I have and I don't have any indexing available than I am going run whatever their record bearing and distance was. The distance probably 80 chains and whatever this is and we will talk about how that works. But the point is what if I had retraced a whole bunch of these lines in my project, and I had found that every time he says he went 80 chains, he was always a little short, maybe 79.90.

That, if you find a consistent pattern of that. That is what we call an index and an index then what I would do was if I was going to then run this line record distance, I would adjust it to 79.90 if that is what I had found was the consistent ratio on these other lines.

So that is an index and you can do it with bearing or you can do it with distance, or both. Now let's just remember something from when you took a statistics class, You know, you want to make sure your sample, using statistic's language, your sample is large enough, is a big enough representation of the population.

Now because when you apply an index, what you are saying is hey his chain was a little bit long or a little bit short. My example was long. His chain was a little bit longer than reality. And so when he says he just went 80 and I don't have anything to proportion against, but I know he went 79.90 on these others, I'll go 79.90. That's an index. But how many lines do you have to retrace to find that?

Now actually I think it was in the 47 Manual or maybe the 1930 or both maybe. It actually gave you some guidance on that and it talked about several miles. I think that is good advice. Now again, you may be finding quarter corners here and so you might have ten or twelve 40 -chain increments where he said he went 40 and he only went 39.95 for example. Well great.


DIAGRAM A full size version can be found in the Diagram section at the end of this study guide.

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But what I am going to say is that when you take a statistical sample of something, it needs to be more than one or two, okay. Because you are saying this sample is representative of everything he did. It is just like these samples that they take for elections.

You know when they interview 1,000 voting adults or whatever you know and they come back and say, well they're for this initiative because 1,000 of them out of that $75 \%$ said; so what they are saying that 1,000 is a big enough sample to represent the rest of the population in your state or the voting district.

So what I am saying here is that you have to be very careful when indexing because you may have run two lines and they are both short and oh well then, he is running short all the time. Well, no. I think that you need to have a bigger sample. All I have to do is find a couple of lines where he is long and I have disproven your index completely. So think about that on that sample size.

Also let's understand that you don't just go take let's say you measure ten miles of this just as a you know, you must have a nice big project. You measure ten miles of this and it is all over the place. This one is long and this one is short this one is right on, this one is right on, next one is short, next one is long, next one is long. So you know you know you have a hodge podge of long, short and right on the record. You don't take those ten miles and mean that or average that. That is not an index.

An index is a consistent pattern of error in the record measurements. Now I am just going to tell you that you know there is not a lot of situations that come up where you are going to index it one of these one point, two point or three point control problems but in my career of 35 years I have never been able even when I have had opportunity to do an index, I have never been able to because I can not find that consistency.

Now I am not telling you that I am the standard by which to measure that I am just letting you know that I have known about indexing a long time and I have looked for it and opportunities for it and I have never found it because I am always all over the map.

Now if you've got ten miles and one of those miles are short and

## Restoration of Lost Corners - Part 3

the others are all long. Throw the short one out and average those other ones. Because you can have one odd-ball in there and again just go back to some of your basis statistics training and think about how you would sample things and how you might throw out the red herring or whatever. But let's recognize that indexing doesn't occur as often now GLO and BLM, they have had over the history you know of the Public Land System they have had tremendous opportunity to index. Why? Because up until just a few years ago, our projects were township at a time.

Well you know when you do the entire township, you've retraced every mile of line in that township, you've got a really good picture of what that surveyor did and how long his or short his measurements were or his angles were always to the right or always to the left. You know his bearings I should say. So you could figure that out. But again indexing is there and I wanted to explain it to you because all three of these point control methods mention it. So I showed you that example a few minutes ago. Let's talk about a two point control.

Two point control is used where the corners, that normally have been double proportioned, have only been established from two directions.

What we do is run record bearing and distance from each of the corners that we do have and we set temporary points. We are going to have two separate temporary points there. You make a cardinal move from each of those true points which means you are going to inverse back to your controlling corners to get your bearings and distances.

What you are going to see is that two point control is not a bearing bearing intersection and it is not a distance distance intersection either.

So once again here is the problem we have a section corner which has been established by the GLO or BLM and the problem is that these lines were never, we have no returns on those.

So we can't do a normal double proportion because a normal double proportion requires us to go, I'm assuming the quarter

## Restoration of Lost Corners - Part 3

corners are lost here, requires us to go between controlling corners each way, but we don't have that option here so you have got to come up with a way to set this lost corner, and the way the Manual says, and you can read about that in 7-14 and in fact it is the paragraph right above the index correction discussion.

It says where the intersecting lines have been established in only two of the directions, the record distances to the nearest identified corners on these two lines will control the position of the temporary points. So there is your record information. Then from the latter, the temporary points, the cardinal offsets will be made to fix the corner point.

What they are telling, going back to the slide, what they are telling us is that I've got to say that this is my found corner down here and let's put a quarter corner in let's not be so unfair to the quarter corners, and so I have to run record bearing and distance from here and I've got to run record bearing and distance from here and they are probably going to come out in two different places and if they don't it's an absolute miracle. So let's look at one of these and see what it is we are going to do.

Two point control is what we have here. And here is your examples of when you use it and that sort of thing. You can have this in completion surveys where it looks like the whole town was done at one time now but it was done in two or three different surveys you remember that from course 2 , talking about completions.

So you want to think about that. How you restore things. Now so here is our situation, I've got a section corner that is lost. It is over in here somewhere and I don't have any quarter corners here so I have a found section corner here and a found section corner here, now the record, I kept it cardinal to keep it simple, the record on this line was east 80 chains, the record on this one was north 80 chains.

I've also given you coordinates in chains that are from your survey. That could be a traverse that could be GPS that could be whatever. You determine coordinates on those positions. So if this corner here is at 100 and 100 and go due east 80 chains, that is going to put me at, because I am going due east it will still be

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down here my temporary is going to be at north 100 and at east 180. Right? North 100 and east 180.

Cause I went due east 80 chains exactly from this coordinate. So that gives me a temporary point down here. Then I am going to go up to this other found existent section corner and I am going to run record bearing and distance from it.

As you can see its coordinates, northing coordinates that you found it to be is at 180.22 north, 179.40 east. So if I go due south 80 chains from that coordinate, I am gong to set a temporary point and that temporary point here is going to be at a coordinate of 100.22 north because I went due south 80 chains and 179.40 east. But, so what we have done is exactly what the Manual said, we have run record bearing distance from the two corners, set temporary points and then what did the Manual say to do? It said to move cardinal offsets, cardinal offsets, well you can do that in math, see.

Because if you are going due north from this temporary point then it is going to maintain the same eastern coordinate as your temp had. So the final section corner that you are setting here is going to have an easting of 180 even. Whereas I am going to do the same from here, this one the temporary point right here, has a northing of 100.22 and an easting of 179.40 well I am going to make a cardinal offset. I am going to go due east from this to where these intersect. See, cardinal offset to where they intersect. Well if I go due east, the northing will remain the same, the easting will change, but I am not going to use the easting for that. I am going to use the northing. Therefore, there is our answer.

So you see what I am saying here is you run the record bearing and distance, you set your temps, you do this all in math, and then you make cardinal offsets, which means you maintain either the easting coordinate here or the northing coordinate here so that it was a cardinal move and that gives you the northing and easting of the corner that was lost that you are resetting. And now if you remember the last step that I had in there was now you will have to inverse from there back down to the two controlling corners so that you actually get your bearings and distances there. And if you look closely at the drawing you will see that this is not a bearing-bearing intersection, because a bearing-bearing

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intersection would have been here right? If the two record bearings had met. And if it was a distance distance intersection, it would have been somewhere over here but not at the same point. But it is not. It is not a bearing bearing intersection or a distance distance intersection, it is a two point control and that is how that is done.


EXERCISE Before moving on to the next topic, complete the "Two-Point Control Exercise" which can be found in the Exercise section at the end of this study guide.

## Three Point Control

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- Do a proportion on the through line, using a cardinal equivalent
- Run record B\&D on the single line (indexing?)
- Make cardinal moves from these two results to set actual point
- Inverse to controlling corners for final courses
Three Point Control
    *
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So let's take a look at that then. Three point control what will that require? You are going to do a proportion on the through line or the complete line that runs through there and that is going to be with cardinal equivalents which unless there is a bearing break, it probably won't matter. We run record bearing and distance on the single line, the line that does not have a continuation to it and you can index on that, if that is applicable or appropriate.

You make cardinal moves from those two points, those two temporary points, once again that sets the actual points of cardinal moves, your basis of bearings is absolutely critical okay and you inverse back to your controlling corners for your final courses.

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Here is a plat. Same plat, just different part of it now. We just did this corner a few minutes ago, two point control. But notice on this section corner, we have three lines coming into it. We have this one, this one and this one. Now see, I can do, let me make us some corners here, let's say all the quarter corners are in here. So I am putting all those quarter corners in. I can do a normal east west proportion, double proportion with cardinal equivalents here. I can do that here.

But going north and south, I don't have anything to proportion to out here. So I am going to have to run record bearing and distance from this quarter corner up to here and I can index that but you can see what is happening when you do the proportion east west, it is going to give you an easting, when you run record bearing and distance up here it is going to give you a northing that is going to be those cardinal offsets just like we saw on the two pointer except this one has three point control. So let's see how this works. In the drawing that I have given you, you can see the record dimensions, I kept everything, but not everything cardinal, but pretty close.

We've got north 31.22 chains there and 8.78 still north there is where this section corner or township corner hadn't been set and then there was a line only run off to the east here and there is our bearing 89.22 east 40 chains.

Now the, so what we just read and what we just talked about as we are going a normal proportion because that is a complete line, the through line. It went all the way through. I am going to do a normal proportion there but on this one I am going to run record bearing and distance. So the normal proportion north south is going to give me a northing, the run record bearing indexed if possible is going to give me an easting, and so we are going to make these cardinal moves to come up with a point. So how does this work?

diagram A full size version can be found in the Diagram section at the end of this study guide.


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On the north south I am going to do just my normal computation like we did before when we take cardinal equivalents. The record is 40 chains, 31.22 plus 8.78 is 40 . It is north here so there is no cardinal reduction. They are cardinal equivalents but they are close enough so that you don't have to worry about it. And then I am going to compare that to my measured cardinal equivalent.

Well I can do that quite easily, right? Because I can take this northing coordinate and this northing coordinate, these are my measured coordinates in chains to the corners, existent corners that I found, the controlling corners, and so I know that I am at 40.88 because that is my difference in northings. So I've got 40 in the record is to 40.88 in the measured as 31.22 that was this one here, is to " X ", " X " equals 31.91.

So just like we did on double proportions, I can take the 31.91 and simple add it to that northing, because I solved this line. So that means that the temp, if you will, that this is going to be at is going to be at is going to be at 131.91, of course that is going to be our northing answer because we are going to make a cardinal move off of that.

So it will maintain its northing. So 131.91 is going to be the northing of this lost corner. And then how do I do the east west? Well, we have a coordinate over here. At this point and we are going to run record bearing and distance there. Now record bearing and distance is 40 chains but if you go 89.22 see we are going to reduce this to cardinal, if you actually ran 89.22 at 40 chains it would, the cardinal equivalent would be less than that.

You would not be 40 chains due east or west, of this point, you would be in the cardinal. You would be something slightly different. And see that is the whole idea, if you run record bearing and distance that is on an angle.

And so your distance, your point here, the easting of it is not going to be 40 chains from here. So just think about it and it will make sense. So what I want to do is go in this case south 89.22 , well I'm going to go north west, sorry, north 89.22 west, right, because we are going back the other way. North 89.22 west, so I have to take the co sine of 38 minutes, right, times 40 chains and I believe that comes out to real close to 40 chains.

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Now you want to always check this, don't assume. But it looks like at least the answer I got that must be right, it comes out right at 40. Because all I got to do is take this easting coordinate and subtract my distance here, not the 40, but the cardinal equivalent, which is probably 40 anyway, and it looks like it is and that gives me 98.98 and that is of course the easting of the corner.

Now a couple of thoughts here while we are looking at this, in the record we had a straight line here and another line that went out almost at 90 degrees. Understand that I have exaggerated here just so that you can see it but you could very easily end up with a bearing break out here, and when you are done, this thing could actually have an angle break in there and you might.

Many a surveyor says well I can't do that because the record says this is a straight line. Well what you are doing if you are going to insist that that be a straight line, then you are throwing out this line.

You are saying that that line doesn't matter or doesn't count. And I want to remind you that in the first hour, the first video lecture, we read some things in the Manual that said that you weigh equally all parts of the line, that you give equal relative weight to all parts of the line, that you don't prefer two corners over the third or any more you weigh all parts of how that corner was established, so if you see on that slide again, the problem is if you want to force this to be straight, and I don't think that that is a good idea.

Because what you are saying is that only the positions of this corner and this corner matter and this corner doesn't matter and this measurement doesn't matter. Yes it does. Under the law and under the way the Manual describes three point control, it does so. It is possible, it is very rare that you are going to have it this heavy of a bearing break, although the more this line is offset from the center of this line, the more likely you are going to have a heavier bearing break at this point.

But it is very likely that it is going to end up with something that doesn't perfectly resemble that but that is because you weighed all parts of the line, you took all three corners and you used the data

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between them to come up with this position, using the book and that is where we came up with that coordinate.

That is one thing that I wanted to mention. One other here, three point control, it doesn't happen a lot. And I want to make sure you realize that and notice this statement down below here, This is not to be used on lost closing corners. There is an awful lot of places where it might look like a three point control problem, but in fact the corner that is missing is a closing corner. We're going to talk about that a little later in this course for lost closing corners. But understand that the three point control only really applies when the corner that you are trying to reestablish has returns coming into it that are all from the same survey.

Well, that may be after, if you are retracing a resurvey then what was a closing corner has now been adjusted to the line barring any major problem there, you might be able to use three point on it then, but really three point is not designed for closing corners. That's the bottom line. So keep that in mind.

Now we are going through these point control methods and there is one more here and that is single point control. It's a pretty easy one. It is over in 7-56 of the Manual. Let me just read that real quick to you. It is kind of you know not the best worded but it is under 7-56.

They call it original control. Original control, where a line has been terminated with measurement in one direction only. That means they ran out and stopped measurement. There is no connection anywhere just that one that you made in one direction only.

A lost corner will be restored by record bearing and distance, counting from the nearest regular corner, the latter having been duly identified or restored. And then they talk about indexing and indexing here as you can see can also apply. Now single point control.

## Restoration of Lost Corners - Part 3

Let's take a look then this is going to apply where the line was established, the corner was established in one direction only.

Those can sometimes be meander corners where the lake was too wide or the river was too wide or at the ocean, quarter corners that they stubbed in, there are some other situations, I'll show you apply here are some others. 7-56 we just read it. It told us to run record bearing and distance. It did say that indexing is allowed, if you can prove it. And as I have said to you, maybe you can't.

Take a look at this plat. Here is a big canyon, impassable. All right, this is up near Sedona, Arizona. And these big canyons have the red rocks down in here.

But when the GLO did the stuff up on the top up here, this is up on a mesa with pine trees and that sort of thing. Notice that they did not complete the sections out into the impassable canyon. Now here this is an interesting one, they started at this section corner, came over here and set a quarter corner, came over here and set a section corner, came over here and set a section corner and then stopped.

And there are no measurements coming into this section corner or this quarter corner, you know going on out that way or something. There is nothing there. So the question here is, how would I reset any of these corners, frankly? You know, if all three of these were lost and this is found, well then you would run record bearing and distance, record bearing and distance, record bearing and distance.

That is what 7-56 is telling us to do. That is the kind of situation we have. Well as you can see on this plat well there are a number of single point control problems on this one as well, okay, and there is another one down there. Here is a two point but that is not our subject. But you see you get combinations of it in these surveys like this. So that is what we are talking about, a single point control and how to set it and again as the drawing showed us, well just run whatever the record is, you can index it, run record bearing and distance.

## Single Point Control

- Used where line established from one direction only (MC, stubbed 1/4's)
- Found at 7-56 of BLM Manual
- Run record bearing and distance......
- Indexing allowed for bearing and distance

dagram A full size version can be found in the Diagram section at the end of this study guide.


## Restoration of Lost Corners - Part 3

Now frankly, with any of these point control methods, but especially the single point control. If I ran here due east 40 chains and there is a fence that is slightly off you know and the fence ends at 40 chains you know I would at least be looking at that as a possibility of evidence because you know indexing with these single point control, who knows what is goes on. Each line is kind of unique. But what you want to be careful of is you don't want to just say well okay here's like they stubbed south okay, you don't want to say well okay I'll look at this half a mile here and they said they went 90 degrees so I'm just going to turn 90 .

Well you know you want to be careful of that and say that that is indexed. That's not. Because you see I could just as easily go and take the half mile at the other side of your corner and turn 90 there if that's the record. And I will miss this. I will miss your point. So you want to be careful with those things. But I do look a little bit more in these single point control situations. Look a little bit more at what's going on on the ground and it may, if you will, allow you to do a local index or just accept, or what I am saying is accept the fence corner as the best available evidence. But notice that I had a lot of "what ifs" in there.

Here's the bottom line, the Manual says run record bearing and distance so if you are running record bearing and distance to a single point control a corner than I am assuming you have not found any acceptable evidence. So in other words you have already looked at the fence and decided it wasn't that good. But that's single point control. That's how that works.

Now we are going to take a break here in this video lecture and come back with another one here in a few minutes and start talking about some things that well the grant boundary method especially which takes us away from the rectangular system and goes into the non-rectangular entities so we'll be talking about that on the next lecture. So Ron and I will both see you over there.


EXERCISE Before moving on to the next topic, complete the "Original Control (Single Point) Exercise" which can be found in the Exercise section at the end of this study guide.


## DIAGRAM






## EXERCISE

## DOUBLE PROPORTION EXERCISE




## N-S Proportion:

Record latitude between controlling corners: $\mathrm{N} .80 .00 \mathrm{chs} .=5280.00 \mathrm{ft}$.
Retrace latitude between controlling corners: N .5302 .69 ft .
Record proportion is midpoint, therefore the latitude to the lost sec. cor. is: 2651.35 ft .
N. $10000.00+2651.35=$ N. 12651.35

## E-W Proportion

Record:
Cardinal equivalent of the departure of the E $1 / 2$ mile: $\operatorname{Sin} 78^{\circ} 26^{\prime} \times 36.06=35.328 \mathrm{chs} .=2331.63 \mathrm{ft}$.
Cardinal equivalent of the departure of the $W^{1 / 2}$ mile: Sin $89^{\circ} 54^{\prime} \times 40.22=40.220 \mathrm{chs} .=2654.52 \mathrm{ft}$.
Record departure between controlling corners: 4986.17 ft .
Retrace departure between controlling corners: 5093.30 ft .
Retract departure $\div$ Record departure $=\mathrm{K}$
$5093.30 \div 4986.15=1.021490$
K x Record departure of each segment of the line = Proportionate departure of each segment of the line $1.021490 \times 2331.65=2381.76 \mathrm{ft}$.
" $\quad \mathrm{x} 2654.52=2711.56 \mathrm{ft}$.
E. $12455.43-2381.76=$ E. 10073.67 ft .

## Proportionate Position of the Section Corner

N. 12651.35
E. 10073.67


## Two Point



## Calculate N-S

Record latitude: $\operatorname{Cos} 12^{\circ} 54^{\prime} \times 2717.88 \mathrm{ft}$. ( 41.18 chs.) $=2649.28 \mathrm{ft}$.
N. $10000.00+$ Record Latitude $=$ Proportionate latitude for the corner
N.10000. 00 + $2649.28=\mathbf{N} .12649 .28$

## Calculate E-W

Record departure: $\operatorname{Sin} 89^{\circ} 54 ’$ x 2654.52 ft . ( 40.22 chs .) $=2654.52 \mathrm{ft}$.
E. $7951.75+2654.52=\mathbf{E . 1 0 6 0 6 . 2 7}$

## Proportionate Position of the Section Corner

N. 12649.28
E. 10606.27
(N.

## Three Point




## N-S Proportion:

Record latitude between controlling corners: N. 5338.06 ft .
Retrace latitude between controlling corners: N .5317 .96 ft .
Retrace $\div$ Record $=\mathrm{K}$
$5317.96 \div 5338.06=0.996235$
K x Record latitude of each segment of the line = Proportionate latitude of the line segment $0.996235 \times 2690.16 \mathrm{ft}$. ( 40.760 chs .) $=2680.03$
" x $2647.90 \mathrm{ft} .(40.120$ chs. $)=2637.93$
N. $10000.00+2680.03=\mathbf{N} .12680 .03$
N. $12680.03+2637.93=\mathrm{N} .15317 .96$

## E-W Proportion

Sin $83^{\circ} 41^{\prime} \times 2373.36 \mathrm{ft}$. ( 35.960 chs .) = Record departure (minus because the line is West) $0.110023 \times 2373.36=-2358.95 \mathrm{ft}$.
12212.32-2358.95 = E. 9853.37

## Proportionate Position of the Section Corner

N. 12680.03
E. 9853.37

| Unsurveyed |  |
| :---: | :---: |

## Original Control (one point)




## Sec. 7-56 Original Control

Where a line has been terminated with measurement in one direction only, a lost corner will be restored by record bearing and distance, counting from the nearest regular corner, the latter having been duly identified or restored.

An index correction for average error in original measurement should be used, if appropriate, as discussed in section 5-29.

## Calculate N-S

(An index correction is not appropriate because we have not retraced enough of the retracement survey) $\operatorname{Cos} 1^{\circ} 58^{\prime}$ x 2115.96 ft ( 32.06 chs.) $=$ S. 2114.71
N.10000.00 - $2114.71=\mathbf{N} .7885 .29$

## Calculate E-W

(An index correction is not appropriate because we have not retraced enough of the retracement survey) Sin $1^{\circ} 58^{\prime}$ x 2115.96 ft . ( 32.06 chs .) $=$ E. 72.62 ft .
E. $10000.00+72.62=\mathbf{E} .10072 .62$

## Proportionate Position of the Section Corner




## HANDOUT

# DOUBLE PROPORTION MADE COMPLEX 

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#### Abstract

Our ever increasing ability to accurately measure make it more critical that we understand the geodetic and legal concepts behind some common survey principals. As presented here, the process of determining the position of a lost section corner under the Public Land Survey rules is well known, and is even generally a matter of actual statute law. On the technical side this paper will discuss how an awareness of the unusual characteristics of the 'Public Land Survey System Datum' and the historical 'Manual' procedures lead to some interesting conclusions about the proper way to compute double proportion positions for lost corners. This discussion will include examples of how both large and small errors can creep into the process when using coordinates, especially State Plane Coordinates, and point out dangerous situations as well as proper methods to use to avoid pitfalls.


## GENERAL

This presentation deals with one of those seemingly insignificant technical issues that may rise up and bite you if you are not careful. This particular discussion relates to the procedure used to restore certain lost corners in Public Land Surveys by the process called double proportion. This is a well known procedure to surveyors in such public land states who practice in suburban or rural areas. It is a process that seems straightforward on the surface, but is also sometimes misunderstood and incorrectly computed.

In defense of this technical presentation, I would like to point out that in my opinion there are many aspects to the profession of surveying. Some of the aspects on which we place the highest importance are the evaluation of evidence, discovery and analysis of prior records and application of judgement. These and other professional issues are well recognized and are significant issues in licensure. Somewhere on the list of attributes that constitute the makeup of the profession of surveying is technical expertise and knowledge of proper procedures in measurement and computation. By no means are these considerations primary, but neither are they insignificant. This discussion is almost entirely technical and one sided. Whereas in the real world I recognize that many other factors control our actions and considerations.

Before I can illustrate some of the technical quirks of double proportion, I need to briefly describe something that I refer to as 'The PLSS Datum'. This datum is simple but has some unique and even strange attributes. A thorough description of it and all it's consequences could easily be the topic of several papers, so what is outlined here is necessarily brief.

## THE PLSS DATUM

The 'PLSS Datum' is the reference system by which the majority of the PLSS surveys are theoretically reported. The data being reported on a BLM or GLO Cadastral Survey plat are, of course, bearings and distances. But bearings and distances with reference to what? The current BLM Manual of Surveying Instructions, 1973 states:

[^0]true north or south."
"2-74. .... By basic law and the Manual requirements, the directions of all lines are stated in terms of angular measure referred to the true north (or south) at the point of record."

Distances. These and other references in the BLM and GLO Manuals make it clear that the frame of reference for distances is defined as horizontal measure in chains based on the U.S. Survey Foot at actual ground elevation. This is of importance when performing computations in projections or at sea-level when the actual lines are at a significant elevation. If you are computing proportions in a projection, the variation of elevation over a project can have a small effect, the elevation difference in essence weights the record ground measurements. This usually is a small effect unless the lines differ in elevation by a 1000 ft . or so.

Bearings: The above Manual sections and others identify the frame of reference for direction as something called 'Mean True Bearings' referenced to the true astronomic meridian '..at the point of record.' For those of you familiar with basic geodesy you will recognize that this is a basis of bearing that changes as you go east and west since the reference meridians are not parallel but converge towards the pole.

Because this is a changing reference, the direction of a straight line on the ground can be described with a forward bearing based on the meridian at the beginning end, or with a differing back bearing based on the meridian at the end point. The difference between them is the angle of convergency of the two meridians. If we want to accurately describe how far north or west the line goes in a geodetic sense, we need to use the average or 'mean' of these two values. This 'mean bearing' is essentially identical to the bearing of the traverse line with reference to it's midpoint. Thus the 'point of record' for determining the bearing of a straight traverse line can be said to be the meridian at the midpoint of the line.

Straight Lines: Therefore, one unusual byproduct of the PLSS datum is that:

## Straight lines on the ground are lines of constantly changing bearing.

A straight line is basically what you would lay out by double centering or projecting a direct line of sight. The only straight line that does have a constant bearing is the meridian or north and south line. An example of a boundary that might be a straight line is one that is described as a straight line running from one physical monument to another. Such a line, if reported in the PLSS Datum would have different forward and back bearings, and different bearings at each point along it.

Rhumb Lines: It is also apparent from the various GLO and BLM Survey Manuals and the actual methods that were used to lay out the public land surveys that most boundary lines in the PLSS are intended not to be straight lines but lines of constant bearing or Rhumb Lines. Such lines cross every meridian at the same angle and are thus curved as viewed on the ground.

Therefore, another unusual byproduct of the 'PLSS datum' is that:

## Lines of constant bearing are curved lines on the ground.

For example, the solar compass and transit were instruments that determined bearing at each setup, and when matched with traditional chaining, measured or laid out lines of constant bearing.

The 'Manual' discussion of latitudinal arcs illustrate one example of a rhumb line. A parallel of latitude is a line that is due East and West in the PLSS Datum. Since it crosses each meridian at a 90 degree angle, it has a mean bearing of East or West. Lines of constant bearing in the PLSS datum will appear curved on the ground. It also turns out that the mean bearing of any chord or sub-chord connecting any two points along such a line is the same as the bearing of the rhumb line itself. Thus it is possible to lay out points on a rhumb line by correcting traverse lines to their mean bearing in computations.

## DOUBLE PROPORTION

Now let's look at the definition of double proportion as stated in the BLM Manual of Surveying Instructions, 1973, which states:
"5-25. The term 'double proportionate measurement' is applied to a new measurement made between four known corners, two each on intersecting meridional and latitudinal lines, for the purpose of relating the intersection to both.

In effect, by double proportionate measurement the record directions are disregarded, excepting only where there is some acceptable supplemental survey record, some physical evidence, or testimony that may be brought into the control. Corners to the north and south control any intermediate latitudinal position. Corners to the east and west control the position in longitude."
"Lengths of proportioned lines are comparable only when reduced to their cardinal equivalents. "
Cardinal Equivalents: The last sentence in the above quote is one that requires some explanation. What it means is that only the easterly components (or departures) of the E-W controlling record lines are used to compute the $E$ and W position, and only the northerly components (or latitudes) of the N-S controlling record lines are used to compute the $\mathbf{N}$ and $\mathbf{S}$ position. This is different than using the line lengths or distances on the record line.

Figure 2 illustrates the cardinal equivalents for some of the lines in the example record shown in Figure 1. Neglecting to correct the record for cardinal equivalents won't usually get you in trouble since most section lines in the original surveys are very near to cardinal and the correction is insignificant. There are, however, many situations in public land surveys where this is not the case. This situation will also occur where a retracement or subsequent GLO or BLM resurvey has reported new measurements in the PLSS datum, and the lines are distorted. The record shown in Figure 1 is a typical example, and is used to illustrate the problem.


Figure 1 - Example Record


Figure 2 - Cardinal Equivalents

## Cardinal offsets

The Manual of Surveying Instructions, 1973 Section 5-26 describes a process for performing a double proportion. The Manual section states:
"5-26. In order to restore a lost corner of four townships, a retracement will first be made between the nearest known corners on the meridional line, north and south of the missing corner, and upon that line a temporary stake will be placed at the proper proportionate distance; this will determine the latitude of the lost corner.
"Next, the nearest corners on the latitudinal line will be connected, and a second point will be marked for the proportionate measurement east and west; this point will determine the position of the lost corner in departure (or longitude).
"Then, through the first temporary stake run a line east or west, and through the second temporary stake a line north or south, as relative situations may determine; the intersection of these two lines will fix the position for the restored corner."

Such a process would probably be impractical in the field if followed to the letter. It is, however, a valuable way to conceptualize a proper solution of a double proportion and a good way to model a computational method.

In brief, the three part process as described consists of:
1 A single proportion using the record $\mathrm{E}-\mathrm{W}$ cardinal equivalents between the control E and W . In the Figure 3 example this would be point 'A'.
2 A single proportion using the record $\mathrm{N}-\mathrm{S}$ cardinal equivalents between the control N and S . In the Figure 3 example this would be point ' B '.
3 Cardinal (true mean) offsets to intersection from those two points. In the Figure this results in point ' C '.

This last requirement can be a problem if you are not careful using coordinates, since to make the offsets cardinal requires knowledge of and proper correction to true north at those points. The common process of using the East coordinate of the E-W proportion and the North coordinate of the N-S proportion is equivalent to making a GRID offset, exagerrated In Figure 3 as point ' D ', which can be incorrect.


Figure 3 -Cardinal Offsets True vs Grid

## Problems with State Planes.

In using state plane coordinates there are several small problems, however dealing with cardinal offsets is the most critical. This is true because with State Planes there can be a large difference between grid and true north, which is called the mapping angle. There are three methods that can be used to correct for cardinal offsets using State Plane coordinates, they are:

- Compute the mapping angle at point A to find what grid azimuth equals true North, then compute the mapping angle midway between A and B to find what grid azimuth equals mean East, them compute a grid bearing intersect. If the offsets are not large, one computation of mapping angle in the area will be adequate.
- Convert the coordinates to latitudes and longitudes at points A and B, use the latitude of B and the longitude of A and convert back to State Plane. This method essentially makes a geodetic cardinal offset.
- Convert the coordinates of the control corners to latitudes and longitudes and proportion them in a similar way to the above.

Small errors can still exist in the computation due to the datum differences. In State Planes the grid scale factor varies over the project. If you wanted to be perfect, this would require you to weight the proportions according to the mean scale factor over each line. This effect is very small. The last method is the easiest, since no correction for scale factor enters into the problem. However, performing proportions using geodetic coordinates directly can still have error if the lines are at very different elevations, since the PLSS datum represents measurements at actual average ground elevation
over the line.
In the example problem, the error caused by direct proportion of the State Plane Coordiantes is in excess of 20 ft . If you used a local grid or basis of bearings the following table illustrates corresponding errors for this example.

```
Grid Angle Error D to C
    1`05' 20"
        0`10' 3.40 ft. Assumed 10' off true.
        0`01' 0.65 ft. Assumed 01' off true.
    0` 00' 45' 0.46 ft. Basis of Bearing, Solar 1 mile E.
```


## SUMMARY

It can be shown that in an ideal world and with a recognition of the properties of the PLSS Datum, the only way to properly restore a corner in its true original position is by diligent application of the Manual procedures, correcting record to cardinal equivalents, proportioning and making true cardinal offsets.

# Course 4: Restoration of Lost Corners Study Guide 

## COURSE DESCRIPTION:

## COURSE

 OBJECTIVES:COURSE INSTRUCTOR(S):

## VIDEO LECTURE

TITLE:

This course consists of four videos, some reading, and three exercises, on the "Restoration of Lost Corners". The legal, mathematical, and practical applications of the methods of proportioning, as found in the Manual of Surveying Instructions, are presented. Students will be able to address what corners control in most situations, how to proportion properly, what legal principles are involved when proportioning, and how to deal with the latitudinal curve. A lengthy discussion of convergence and curvature in the PLSS is also included.

Upon completion of this course, students will be able to:

- Define the three corner conditions listed in the Manual of Surveying Instructions
- Describe, identify applicability, and compute proportions using all methods
- Demonstrate an understanding of curvature in the PLSS

Dennis Mouland, Bureau of Land Management
Ron Scherler, Bureau of Land Management

Restoration of Lost Corners - Part 4 (76 minutes)

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## Restoration of Lost Corners - Part 4

## Introduction

Welcome back to this time Video Lecture 4 of the Restoration of Lost Corners course. A lot of information here, isn't there? And we are moving along as quickly as we dare go. We have been talking about things, single and double proportions and various modifications of those in the Public Land System.

We are now going to take a step away from the PLSS and talk about the grant boundary method. It is a method which is applied to what we call the non-rectangular entities. Things that were created outside of the public domain but are not part of the Public Land Survey System in other words exceptions to the rectangular grid that came about in the 1785 Act.

And of course some of those are because they existed even before the 1785 Act or they existed at least before the Public Land Survey System got there like the Spanish Land Grants and others were things because Congress created sales and exchange authorities that had certain limitations on where the boundaries could and so because of those limitations, we have boundaries that don't follow the Public Land System.

Good example for those of you in the western third of this country is mining claims, you know lode mining claims by law follow the vein of the load or the vein of ore and those don't go down the section lines or the $16^{\text {th }}$ lines or anything else. They are an odd shape. So we have this method that was designed, I don't know if it was designed, it has evolved over time I am sure but it is what we used to address these.

Now I might mention, before we go on, I just might mention you know there are a lot of non-rectangular parcels especially in the colonial states, it all is, the government, or the Indians, the tribes have acquired and I would not be real quick to apply the grant boundary method because I think in some of those states you may have regulation on that but on most of them I would think the compass rule might be a better deal but I am going to show you the grant boundary method because that is the recommended method and then Ron is going to discuss for a few minutes comparing the grant boundary method and the compass rule and I

## Restoration of Lost Corners - Part 4

think that discussion will help you a little bit more in deciding sometimes when one of those might be better than the other even though the Manual calls for one.

So we'll get to that here in a few minutes.
But the grant boundary method let's take a look at this slide and see what is going on with it. It protects angular relationships at lost corners that is its goal. It applies to corners of most metes and bounds parcels that are within the Public Land System or what we call non-rectangular entities.

It is not applicable to straight line proportions on such parcels. In other words you may have a non-rectangular entity but it's got a five mile straight line and there is a point every 40 chains on that straight line you would use regular single proportion there this is only for where we have angular relationships in these nonrectangular entities.

And let me give you a few ideas of some of the things in the public domain that you probably would use the grant boundary method on.

Here's a list. Indian Reservation boundaries, that's the exterior boundaries. Now obviously if that reservation boundary goes down a township line or follows a creek or river or something well than it is probably a different circumstance but if it's a surveyed boundary you know than that would be it. A military reservations.

A lot of those in the west and the mid-west, odd shaped parcels usually. Homestead entry surveys in the western states on National Forest System lands. Small holdings claims which exist primarily in New Mexico. Donation Land Claims which exist in Oregon, a little bit in Washington and I believe some in Florida. The Spanish Land Grants are the Mexican ranchos. They are the same thing but they called them ranchos out in California but they are large parcels of land, usually large that had been deeded by a former government. Some national park boundaries, not all of them, appear in the northern Arizona, the Grand Canyon National Park. Some of it follows section lines, some of it follows mid

## Restoration of Lost Corners - Part 4

section lines, some of it follows canyon rims, and other places it follows odd shaped surveyed lines which are actually monumented. So you just have to look at your situation.

Townsites, some of them. This is under the Townsite Act which allowed the government to sell land for certain community purposes, lighthouse tracks, US surveys, most US surveys are in Alaska. We do have some elsewhere by they are metes and bounds surveys. You can apply the Grant Boundary Method to mineral claims but only when they are isolated and they are not our subject today, we are not talking about these non-rectangular entities all that much especially mineral claims but understand Grant Boundary may not be the best unless it is just an isolated claim.

And independent resurvey tracts you will remember from course 2 , we talked about independent tract resurveys and how they tracted out land and that sort of thing to protect it in their original position so those are some of the places that you would apply the Grant Boundary. Now here let's list here the order in which we do one of these Grant Boundaries.

First of all as with everything, locate your existing corners, find what you do have, tie them in by a traverse and the main reason for this is that you are going to inverse between the two controlling corners that you have found and see their relationship.

Your survey should be based on true bearing, as always. We are going to compute a rotation and a scale based on what the Manual calls a connecting line. You and I would call that an inverse. And I'll show you that here in a minute. Apply the rotation to the bearings and the scale to the distances and then if you have another set of corners that are lost somewhere else on the parcel, that would be a completely separate set of calculations so that is what we are talking about here now.

If you look at 7-54 in the Manual you will see the discussion of the

## Grant Boundary Method

- Locate existing corners
- Make ties by traverse
- Survey should be based on true bearing
- Compute rotation and scale on connecting line
- Apply rotation to bearings, and scale to distances.
- Separate calcs for next lost set Grant Boundary Method and Figure 7-10 in the Manual there is a diagram there and it is pretty self explanatory but I want to just mention a couple of things that are kind of unique to BLM at least the way the Manual says it. Notice again see point A and point B are the two existing corners that they found in this example, Figure


## Restoration of Lost Corners - Part 4

7-10 in your Manual. Those are the ones that they found.
You can run the record data which you have to do to compute corners of where they should be for search and that is all of these sub T corners here, B sub T, J sub T, whatever and then and on, and then you see is where they actually found B is different from where is should have been as far as the B sub T , that is the temporary based on the record. Now what they do is inverse from point $A$ to the real point $B$ and from point $A$ to where point $B$ was in the record, B sub T, in other words. You get two different inverses.

Again the Manual calls these connecting lines as you can see here. And notice also that the Manual refers to record which you and I would refer to but you and I call it measured, the Manual here refers to it as actual. So just so that helps you understand this drawing you are looking at there on page 143 so and that is basically a list of things we just saw as to what it is we are going to do to make one of these happen

Now we are going to then let's do one and talk about it here, we are going to look at a Grant Boundary Method on a Homestead Entry Survey. This HES has got, that's a Homestead Entry Survey that, those are in the National Forest, but don't worry about it, I am just using an HES as an example, this could be a small Indian Reservation for that matter or whatever. But we found this corner and we found this corner, but these two are lost.

So this is a non-rectangular entity in the public domain so we are going to use the Grant Boundary Method to proportion this. So the way the Manual says to do it, you would start at an existing corner and you would run record bearing and distance, set a temp;
 record bearing and distance, set a temp; record bearing and distance, set a temp and then it wouldn't hit this unless just by miracle, right?

So what you would do is then measure, you would, from the record point you would inverse over to here and then from the point that you actually found you would inverse so you would have two different inverses and that is those two connecting lines that the Manual was talking about. And you are going to take the data on those two connecting lines and you are going to compare

## Restoration of Lost Corners - Part 4

them.
You are going to compare what it is between the record and the measured. And if the measured is longer than the record than you are going to come up with a scale factor that is greater than one. And if you come up with a measured that is shorter than the record, then you are going to come up with something that is less than one, 0.9 that's just a scale factor.

In reality folks, this is just a rotate and scale which many of you even have software that will do that and what I have done is in coordinates, you just pick one that you are going to hold that's what point A is in that corresponding diagram in the Manual and that becomes your record and your measured coordinates. And then I just put in the record and sees where it comes, inverse back, then I have shot this with GPS or whatever, you know traverse and I inverse that and those are my two that I compare.

And so that is why in this diagram you have here there are two coordinates on this point, there's one that is the record, that is where the point was supposed to be in the record but this is where you actually found the evidence on the ground, so we just rotate and scale here, and the scale I have already explained how this distance of this line is what establishes that and the rotation is also what establishes that.

If the measured goes this way and the record goes this way, there is going to be a rotation here. And remember that you are always going to go from record to measured. So if you are going from record to measured, that helps you realize which way you have to rotate.

## Restoration of Lost Corners - Part 4

Now this is that exact same HES that we were looking at on the previous slide. But here I have gone ahead and put in the information.

Now I had already given you your record bearings and distances and a record measured coordinate there and record measured coordinates, so you have all of that information. So I have essentially done the record traverse around here for you by providing you with that coordinate.

So what we want to do here is, we make the inverses and when you do that you are going to find out that the measured line is considerably shorter than the record line and so when you divide those that is going to come out less than one, so it gives us the scale factor of 0.8677 . Now if you are doing this by hand, now

Grant Boundary Method
 again software might do it for you, but if you are doing it by hand, you are going to apply that factor to each of these distances, to the 12.27 here, 4.9 chains there, 10.42 chains there.

So you have now shrunk, if you will, because the scale factor is less than one. You have now shrunk that and all of the distances on there and now we are going to rotate the bearings. Now in this example the bearings are rotating 4 degrees and 30 minutes counter clockwise, so that way. And so you have to pay attention to that right, because we are working in quadrants. So let's look at that then so what you would then do is take the record bearing, north 492730 here and you are going to counter clockwise rotate it so it's going to be less than that 4 degrees and 30 minutes. So what you have down here in this red box are the adjusted bearings and distances using the Grant Boundary Method.

All of the bearings have rotated counter clockwise, all of the distances have been reduced using the scale factor up there. So that is the Grant Boundary Method. Really it is quite simple. It seems odd at first why they would even do it but it is the scale and rotate what you and I would call in the private sector. Now there are two other things that we want to look at on this slide just for the moment. You notice that these two corners down here are closing corners. We are going to talk about how to deal with closing corners on a non-rectangular entity like this a little later after we actually talk about closing corners themselves. So but I wanted you to notice that there because if see, as you can see here

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on this drawing that we have got one of them that's lost, this one is found, but the other one is lost.

You would not want to use the Grant Boundary Method on these and I will show you why later after we talk about closing corners themselves. Also at the very bottom of that slide there is a reference there for you in the case book. You have the case book on your resource CD. The case book folks is just a fabulous reference, an educational thing. We are going to use it for more of our advanced courses and our continuing ed, but we put it into your resource package so that you would have it.

This makes reference to page 89-1 through 89-8 and you might want to take a look there because it is comparison of the Grant Boundary and Compass Rule on an HES just like one of these and it fits in with what Ron is going to talk about here in a minute. But I wanted to let you know that that reference is there but also to make sure that you realize that the case book is a fabulous tool you can go a lot further into some subjects with it.

We just rarely make reference to it here in this course because we just wanted you to have it and we'll use it later on in some other things. So that is the Grant Boundary Method of and by itself. Now where we are going to go here now is that Ron is going to discuss for a few minutes the difference between, as I told you what was going to happen, the difference between the Grant Boundary and the Compass Rule and some of the situations you may want to really think about which one you are going to use and where you might apply it under different situations.

## Compass Rule v. Grant Boundary Method

There are just a couple of things about the Compass Rule and the Grant Boundary that I would like to talk about. You have explained how we do them and what they are used for but there are a couple of issues maybe we need to talk about. First of all, the Manual gives us direction on to use the Grant Boundary for lost corners on a reservation or a Grant Boundary where we have some kind of an irregular type boundary, we have angle points, boundary breaks and really that applies to any kind of boundary that is a non-rectangular boundary, such as mining claims, maybe

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US surveys, Homestead Entry Surveys, some of those kinds of things.

So generally, we are going to use the Grant Boundary in those situations but I want to take just a little time to compare what a Grant Boundary does compared to what a Compass Rule does and see maybe that there is some application there for the Compass Rule. In practice in the BLM normally we will compute corners, lost corners, along these kind of broken boundaries, irregular boundaries. We will compute those using both the Grant Boundary and the Compass Rule and then determine which method may apply or give us the best correction. So let's look at that a minute.

What I have here are just a few diagrams. They are graphic. There are not many numbers but let's take a situation where this is the record and we have a line that goes north 40 chains, then east 10 chains, then north 40 and we have two missing corners in there. So the original record is what you see here. We have a couple of lost corners. Let's see what happens when we use a Grant Boundary and a Compass Rule to do some adjustments here.


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So let's first look and say that we find that original corner, so we have both original corners, but there is an error of ten chains in easting between these two corners. An error of ten chains in easting. North south they are perfect. Now of course we are not going to find that in practice. But let's just look at that.


Now let's see what happens when we adjust these with a Grant Boundary and a Compass Rule.

Remember the Grant Boundary holds the angles so your record angles do not change and what we see here is, look at what happens to that east west line, it now has a fairly significant bearing change along with of course the north and south lines. But look at the Compass Rule remember that there was no north south error, so there is no north south correction to that east west line in the compass adjustment, therefore, it ends up being exactly east west.

It adjusts the angles and the distances and so you end up with a different shape and sometimes it can be significant. There is an example in the case book that is just a real great example. And we
 haven't talked about the case book a lot in this study but or in this course but it is a book of survey examples done by the Bureau that just give all kind of great information and insight and in the book there is a great example of reestablishing a Grant Boundary where the Grant Boundary and the Compass Rule give significantly different answers, significantly different answers. And it is very very clear that one answer is much better than the other, based on information on the ground.

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Now here we have a situation where the error is exactly north south, there is no east west error. The error is exactly north south.

And when we do these two adjustments look what happens. The Grant Boundary appears to give us a fairly good adjustment. It holds the angles. That east west line has a pretty minor bearing and I believe that when I actually calculated this it came out within a degree of east and west. Whereas the Compass Rule because is has all north south error, it applies a fairly significant north south correction to that east west line. So we get a significantly different shape whether we use the Grant Boundary or the Compass Rule. Now what we need to do, so how do we choose which one is best? Well sometimes you can't.

Sometimes you can't. But often you will find old fences and maybe you don't have fence corners, but you have some old occupation lines or fences or roads or something and one method the lines appear to follow those roads or fence lines or lines of
 occupation and the other method doesn't. So you may have topography that fit better with one method than the other, you know, you may have a corner out there and you really don't know where it came from, you don't know whether you can accept it or not accept and one method seems to fit that and one doesn't so you may end up actually accepting the corner once you have done all of this adjusting.

But what I want you to see is that these two methods can give you fairly significantly different answers depending on the shape, the magnitude of the error and the direction of the error and normally within BLM at least we will calculate it both ways and then look

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at all the evidence on the ground, any other evidence we have and then decide which method is best. Now because the Manual says the Grant Boundary is the prescribed method, that is the method that we are going to start with and we are going to have to document why we would go with a Compass Rule rather than a Grant Boundary. But I think that this, it is important to know this and to approach these kinds of surveys in this way. So Dennis with that, we will continue on with your session. Well thank you, Ron, that was a good discussion on that subject.

## Meander Lines

Now we are going to change gears again and we are going to go to something else that is in the Public Land System but is used to help identify water boundaries, usually, and that is the meander lines. Now you will remember, I hope you do, I hope that you will recall that in course number 2 I talked about meander lines a little bit. And I want to remind you that meander lines are not fixed and limiting boundaries with few exceptions.

Now you find meander lines mostly along water boundaries. That doesn't mean that they are automatically navigatable or any of that. We talked about that in course 2 . But you find them there, occasionally you will find them at the base of rugged mountains that sort of things where they wanted to limit the area that was patented and that would not include stuff up a cliff or up some big you know face of a mountain or something, so occasionally you do see those. And that is really one of the questions that comes along.


EXERCISE Before moving on to the next topic, complete the "Grant Boundary \& Compass Rule Exercise" which can be found in the Exercise section at the end of this study guide.

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A meander line was never meant to be a boundary. But there are certain situations where it does become one.

Because just ask the question, how can you have non-riparian meander lines? And really there is a couple of ways. One is it was never riparian to begin with. It is the edge of wastelands and that they were meandered and you will find that. The first time I ever saw that was on a job against the mountains down just south of Albuquerque, New Mexico. But there is another way and that is that it was riparian but the mean high water line has moved away.

Now you will learn in the water boundaries course as well as you put that together with what you heard in course 2, and you realize the mean high water line is if it is a navigatable body of water, the mean high water line is probably the boundary, not the meander line. And so if the mean high water line has moved away to a different place either by erosion or accretion, then what we have is the old original meander line is no longer riparian. That is the best way that I can explain it.

Frankly, in the Manual, you will find this at 7-53 and it is sometimes called the broken boundary method. That is not really a great name for it. I understand why they did it but if you read that third paragraph there, which I won't read to you now, but if you read it you will realize that this is the Compass Rule. This is the Compass Rule exactly. No modifications to it at all. This is exactly how you and I have learned to do the Compass Rule in the past. And so that is basically what they do. Now the strange thing is this says angle points of non-riparian meander lines, and yet you know most meander lines are riparian or at least began that way. And I don't know the whole history of this, I just know that we use this method to adjust any kind of a meander line, whether it is water related or not and again most meander lines are water related.

Let's just realize that here at 7-53 that is what they are talking about. It is the Compass Rule. If you have a Compass Rule software, you want to use an open-ended traverse approach or application on your software because some software has to know if you are coming back to the same point or not. Now without going into too much, again your water boundary course is still

## Non-riparian Meander Lines

- Never meant to be a boundary
- How can you have non-riparian meander lines?
- 1. Edge of waste lands meandered
- 2. MHWL has moved away
- Found at 7-53 in BLM Manual
- Actually the compass rule, using an "open ended traverse" approach


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ahead of you. You know I just want to say that here is the problem when you have meander corners that you have reestablished which would be almost always single proportioned on the section line or if they were stubbed out single point controlled and then you or you found them, either way, you reestablished them or found them.

Then when you connect them, you know the old riparian or nonriparian; the meander lines were often not run with near the precision that the rest of the survey was done. In fact, there are some old references to reading steady for the distances in some areas of the country. So you know, of course think about it, where were most riparian meander lines, they were run down right near the high water and some of the heaviest brush and surveyors took shortcuts and they were kind of sloppy and then chains kind of zig zagging through the brush or whatever or even if they are guessing the distance. But here is the problem, you have a meander corner that hits the river here and another one hits the river here and they've got ten bearings and distances along the edge of that river or lake, whatever and when you connect these two, it will not close.

It is usually going to be, I won't say disastrous, but usually a significant amount of difference between the record and the measured. And so before you can use that meander line, you have to adjust it to force it to close. And the Compass Rule is what we use and the reason you might be doing that is because you might have a mid-section line coming in, well I shouldn't if that's the river, a mid-section line coming in that is going to hit that and you have to compute where it hits the unjusted meander line. More about all that later.

But now let's understand that if you have a meander line that you have to restore, you are going to use the Compass Rule between existing found meander corners, okay, or reestablished meander corners. So the Compass Rule. And that is all it really is. A very simple process. And again, you've got software for that. If you don't know how to do the Compass Rule or don't understand it, we are not going to teach that here.

That is in our minds something that was in elementary surveying and so if you are not familiar with it, get yourself familiar with it.

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But most of you have software that will do that. So that is for adjusting meander lines, Compass Rule. And that makes a lot of sense if you understand how the Compass Rule works. Now I promised you a little earlier that we would talk about lost closing corners and we are going to do that now. What do you do with a closing corner?

## Closing Corners

Well we want to remind ourselves what a closing corner is for to begin with. 7-41 thru 7-49 in the Manual talks about closing corners and let's review what their purpose and their origin is.

In fact, let's just read it. 7-41 thru 7-49. It's kind of funny because you know Chapter 5 and 6 of the Manual are about lost or obliterated corners, right, and yet 7-41 thru 7-49 which is talking about lost closing corners is also where you find the information on what to do with a found closing corner, it is an identified closing corner. So in 7-49 the third paragraph, a recovered closing corner not actually located on the line that was closed upon and if you remember from course 2 that is almost always, will determine the direction of the closing line but not its legal terminance. It only identifies the direction of the line see so the correct position is at the true point of intersection of the two lines and we talked about that in the previous, a couple of courses ago.

However, this is just to remind you that the purpose and origin of closing corners was to not set the actual point, not to go to the trouble and expense of running the senior line but to just make a guess as to where that was and we'll come back and adjust it so most closing corners are either across the senior line or short of the senior line. It is very rare that they got it right on. So it is short or long and what the Manual says well when you have a found existing closing corner, recovered one as they say here, then you use that for the bearing of your junior line, but the true point, in other words, the true section corner, if this were a section corner, closing corner, is at where the two lines actually meet. Now that is just to review what they do.

When you have a closing corner that is lost, you don't know where, it is gone. You have evidence perhaps on the senior line, well you're going to have to determine it somehow. You've got

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evidence on the senior line but you don't have, you may the quarter corner to the south but you don't have the closing corner. How do we re-establish that? That is the first paragraph of 7-41 which says, "A lost closing corner will be reestablished on the true line that it was closed upon," so that is what you and I call the senior line. "And at the proper personal interval between the nearest regular corners to the right and left." So single proportion on the senior line, now I've got a question.

Do you see how that kind of violates a principal of proportioning? Think about it folks. When you have a lost closing corner, it was one chain that came up from the south that set the closing corner and measured the offset to one of those senior corners, but you have a completely different chain that measured between the senior corners on the senior line and the whole concept of proportioning is to lay your chain down against their chain and unfortunately the way closing corners are designed and the way they work, and I am not saying this is not the way to do it I am just saying that it is an odd situation because it violates the basic principal of proportioning and that is to lay your chain down against their chain. Because you lay your chain down to the wrong chain. If you are not familiar with what I am saying there, I'll show you here in just a minute.

Now as I made reference a few minutes ago, even on a nonrectangular entity, if you have a closing corner you should not replace it with the Grant Boundary Method rather you should do just what we are talking about here because it will because a gap or an overlap in the record.

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Now I want to just show you. Remember, here is a Homestead Entry Survey, this is a real one. A real plat. This is a piece of private land up at the Grand Canyon that is known as the Community of Tusean. And these corners down here are all just regular old HES corners, you know, they're, it is a non-rectangular entity, yet the ones up here both, and I even after I blew this up it is kind of hard to see it, but the letter cc right there and another letter cc right there meaning that they are closing corners. Now what does a closing corner telling you? It is telling you that the intent is that this line is junior.

And the intent is to have it abut the senior line, to be right on the senior line, not across it, not short of it. It is a closing corner. It may not have actually been set on the line but they are telling you their intent was. You see the problem is if one or both of these became lost, then and you do a Grant Boundary Method. The Grant Boundary doesn't know anything about the senior line. The Grant Boundary Method is just going to do its thing and so what happens is that you will end up with this.

Where they specifically told you that this was a cc, a closing corner, and this was a closing corner, they wanted to be on the senior line, yet if they're both lost, if you do the Grant Boundary, you are going to do this as it shows here or it might turn up like this, or it might turn up like this.

You know it just depends on how you find the corners or it might turn up the opposite of the way I've drawn it here like that. There is you know an infinite number of ways it might show up. Notice that the whole purpose of them telling us that there was a closing corner was so that there would not be a gap or an overlap.

So if you use the Grant Boundary Method even on a nonrectangular entity to proportion in a closing corner, you are going to violate the intent of the plat. And then here with this and that previous example, the intent was very obvious that they wanted to close against that section line so if you can't find those two corners or one or both, whatever, you are going to have to proportion on the senior line to put it in and make sure that they land on the senior line. In other words, don't use any data down here or here to put that in because you would be doing a Grant

diagram A full size version can be found in the Diagram section at the end of this study guide.


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Boundary, same thing would happen with a Compass Rule by the way, so you don't want to use either one of those, when you have a non-rectangular entity that has a closing corner.

Now so let's do a lost closing corner situation. You see on the slide there, lost closing corner, single proportion on the senior line. That is what is said to do. So here is our situation. We found this quarter corner down here. We found a standard corner on a standard parallel. We found another one over here. They in the old plat said they were 40 chains apart. I have measured them and found them to be 40.22 . So it is a little bit long. I've got to put in this lost closing corner. And if it is truly lost, then I have no idea where it should be here. So what did the book say?

It said you got to put it on the senior line which will be defined by those two corners and you are going to have to proportion it between the nearest corners to the left and to the right which of course is on the senior line. So what I am going to do is set up a proportion between the 40 and the 40.22 and compare that to the
 3.75. Now you see why what I was mentioning earlier how this violates proportioning because the chain that measured the 40 is not the one that measured the 3.75 , yet we are going to use the 3.75 to set this back and we are going to use the 40 .

The chain that measured the 3.75 was the chain that came up this line by a completely different surveyor and completely different survey, usually. But that's what the Manual says, that's how we do it. Closing corners have always been a pain in the neck and always will be. Wish we'd just eliminate them. But even if we did eliminate them, you would still have to deal with all the thousands and thousands and thousands of them that already exist. So there's my ratio, even though it is to the wrong chain, I am going to compare my 40.22 to that chain that said 40 chains across there. So the formula is going to be 40 is to 40.22 as 3.75 , because see that is my record for the short distance, so as 3.75 is to " X ".

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So take a look one more time, I've got that solution on the slide now 40 chains record is to 40.22 in the measured as 3.75 is in the record, all right, is to " X ". " X " equals 3.77 . Obviously, " X " is going to get bigger because 3.75 see the measured here was larger than the record so this 3.75 is going to proportionately go up. So that is what that 3.77 is. And that is how you would set that corner based upon what the book and the law says. On this line, single proportioned on that line, but you adjust your tie that was made by the surveyor to the south. You adjust it based on those pieces of information. And that is a lost closing corner in a nutshell.

So it is a, you know, when you find a closing corner it is more complicated than if you had ever lost one. Frankly, you know you find one, you have to adjust the, you know, get the true intersection to come up with it as we just read there in 7-41.


Whereas, when you don't have one while it is just a single proportion, it is not the greatest solution, frankly, as far as the principles go but it is the solution that we use and I don't know one that would be better. I don't know that it would be all that better to go the 3.75 if you look back on that slide you know. This is something that a lot of people mention. They say well why don't we just put it on the line but we'll just go the 3.75.

Well, you know, the smaller that distance is the safer that probably is. But you know if that distance is a lot longer, say it was 13 chains, well then you know you are going a record distance with that on adjustment. Another thing to remember is that 3.75 , you know, I get a little nervous about it, because you see the closing corner was either up here too long or too short. Right? Now I am exaggerating for scale here but understand that whether he was short or long, the 3.75 is actually a hypotenuse that he measured there. So if you go the 3.75 you are probably going to move the corner further east than it really was unless it is a really short distance or the closing corners extremely close to the line.

You know just running the record distance there is not a really good solution unless it is really small. But when we get to the special cases discussion, we will point out one situation where even with the closing corner you would step away from what 7-41 thru 7-49 says and perhaps do it that way that I was just talking about. So there are always exceptions. Right? That is why this is

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such a great profession. That is why we are doing this training for you. That is why you are taking the training.

## Interiors of Sections

Now the next portion of the course we are going to discuss the setting of lost corners on the interior of a sub-divided section, a section that has already been sub-divided. Ron is going to join me in that discussion.

The idea of and there are a lot of places where BLM has been in or even private surveyors for that matter, have been inside a section, they have sub-divided the section properly, you know as far as computation, method and all that. But some or all of the corners inside of the section have become lost. What does the Manual say about that?

Well, you know the Manual really doesn't give us any direction.
We have some principles laid out in the Manual about how to reestablish corners and it talks a little bit about you know to find out what, how, what method was used to set it. But if we just look at the center quarter of a normal section that was established at intersection of center lines and that corner is lost, there is no prescribed method. And you know you might argue that it should be reestablished at intersection and I could argue that it should be reestablished at double proportion. And if you argue double proportion, I could argue intersection. There really is no prescribed method. But if you step back and think about what is our purpose.

Our purpose is to put the corner back where it was originally. What this really does is give us some leeway I think in determining, looking at all the evidence and deciding which method is the best method. Of course, if it is more recent surveys, hopefully the record information is very accurate. And we are going to get, no matter which method we use, we are going to get a very good solution. Some of the older ones we may not. And we also have to be careful how the corner was set you know because of the three mile method. That's right. We have to be aware of that. That is a single proportion situation. But we have a real mixed bag. For those of you who don't know what a three

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mile is, you will get that in the section subdivision course that comes later.

But just a point that there are a lot of exceptions to things so here we have the leeway, yet you still have to pay attention to what the record says. Yes, yes, we really have to look at the record. So the bottom line here the Manual doesn't address this directly but let's go to the slides to see that there are really two possibilities of how we, at least the Bureau, has does this. And neither one of them gives you an absolute answer anyway.

But one the section was subdivided per the Manual and in fact you know the center lines were run, and the center lines of the quarter sections were run or whatever and the notes would show you how that was done. And the other possibility is that the section was subdivided, they went around the outside of it. But then they subdivided as a traverse. They did not leave all of the controlling points. And there is where you may use the Compass Rule, or distance distance, or Grant Boundary or two point to help solve those. Now you know the bottom line here with many things you are within a section it may be more obliterated than it is lost especially if the corners that they set are really close.

Let's take a look at these examples here. As you can see here on the left, this is the first of those two examples, where this is one section all right and where they have run the center lines of the section just like they are supposed to and set the, you know in a normal section I should say, and they have set the center quarter and then let's assume that this is a normal unlotted section so these were all placed at midpoint. And then this was set at bearing bearing intersection. Now what we are saying here though is what will happen if one of those becomes lost? What if this became lost? The center south $1 / 16^{\text {th }}$.

We have the center of the section, we have this. Well, it was set at midpoint to begin with and you may want to just proceed with that again as it is lost. But remember that you also have a distance that was going out here that was done at the exact same time by the
 same surveyor that measured this and set it. So you have additional information that you may want to consider. The same thing here. Let's just use the west $1 / 16^{\text {th }}$. It was set, we hope, at bearing bearing intersection between those points. Right? Now

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how are you going to reset it if it is truly lost? Do you want to do bearing bearing again? Or as Ron was mentioning with center sections, the same thing could happen here you know.

A double proportion considers all four of these lengths here in a computation that relatively weights the line. So we don't set a center of section the first time by double proportion or this one in a normal section. But what we are talking about is that one or more of these interior corners have become lost. And Dennis, really we don't set section corners by double proportion. That's right. We don't. One of the things that I would like to point out is that on that south $1 / 16^{\text {th. }}$ Many of our prescribed methods look at sort of a higher anarchy of lines where the north south center line takes precedence over the lower subdivision lines. So it might very well be appropriate that the initial method here or the prescribed method would be single proportion along the north south center line.

Always making sure that that is a good answer given the position of that south west $1 / 16^{\text {th }}$. But like we said, there isn't a prescribed method. Now what would you say, and maybe we can go back to the slide for a second, Ron, given the same circumstances, I'm just curious now, let's say the center section is there, south quarter corner is there. This is lost, but there is a $1 / 64^{\text {th }}$ corner or a $1 / 256^{\text {th }}$ corner, does that start to change how you might consider these possibilities? I think it does because I believe the closer the control, then there's less likely to veer. So if we have something very close I mean a $1 / 256^{\text {th }}$ corner, you are talking as close as some bearing trees.

Yes, I think the locations of those other corners have to be looked at and I think that is why we don't have a prescribed method here it is because there are so many variables. We have a prescribed goal and that is put the corner back where it was originally based on the evidence.

Now then considering this other possibility, and looking again at the slide here, this is where the Bureau or a private surveyor for that matter, the quarter corners are in, you know we'll assume that they are all there or can be reestablished, and although they computed it this way. I didn't draw it very well, sorry. They did not monument a center section. They didn't monument the $1 / 16^{\text {th }}$

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corners out here on the section line and all that was actually left was this series of corners that just went down this line.
Let me show you all an example of that, here is a plat, this is a section here in Arizona. Now they did set the 16 ths and the center section here, all right. But look at these little corners zigging and zagging down through here.

dagram A full size version can be found in the Diagrams section at the end of this study guide.

That was a traverse down and my point being you don't have these other points out here in order to do some kind of a you know let's say bearing bearing to if that corner, that would be what the north east south west $1 / 64^{\text {th }}$, if it was missing, and so this is kind of the other situation we are talking about. And maybe this is even more so what you are saying about how close the control is. Yes. And again. What is our goal? Our goal isn't to put it where It should have been, our goal is to put it back where it was so we need to find the closest control, the best control, weigh that with evidence on the ground, fences, use lines, all of those kinds of things and determine what method is going to put it back where it was and what method is best.

Here you know a two point might very well work, you might end up with a Grant Boundary, a Compass Rule could work, you have a lot of options to look at to decide what is going to do the best job of putting that back where it was originally. Yeah and I've seen some of these that were stair-stepped like this that were actually $10 / 24$ th corners. So now you've got these corners that are just a hundred, 200 hundred feet apart and maybe you just go distance distance again you know like you said. If they were

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bearing trees that's what you would do. Yes. Yes. Just swing those in and put it there. And I think hopefully in these kind of situations, we have very little error that we are working with.

These are newer surveys. Newer surveys, yes. And the corners are closer together and so hopefully we are not dealing with a lot of error out there. I could see even a situation like this if we did find one corner, you know one of our control corners were significantly out of position, it might not be very good control for putting a corner back so, yeah, judgment, gathering all of the evidence I think is really important and looking carefully at the record.

And one more thing then going back to the slide you know the bottom line of what we are saying with this kind of things is that it is almost crazy for you to go out and set that $1 / 64^{\text {th }}$ corner and that one which the BLM never set and that one and that one and to then to try to force this to be at that, because you know there is a different set of error, if you will, or issues as they stair step down through there, so really the bottom line with this second situation when the normal control wasn't set that you really think about what is the best, you don't say this corner is missing, what's the best way, I doubt now there may be, it might be tight enough here that its fine, but I doubt that going from here to here, from here to here, bearing bearing intersecting it and then putting that at midpoint.

I really doubt that and I think you to need to use the distances that are actually returned on the plat so if this one was lost, I would be looking at ways for those two to somehow give me a good solution. Yeah, and when you think about all the other reestablishments that we do, they are all based on measurements between monuments. That's right. That is a principle to hold here, measurement between monuments. Not where it should have been, where it was based on the measurements reported in the record between the monuments. Yeah, it is an interesting issue, it's an interesting issue and you have to approach it with the same principles that we use on restoring other corners, I think. I think that's right.

## Restoration of Lost Corners - Part 4

## Special Cases

With that we are now going to switch gears to really our last segment or last sub-topic here under restoration of lost corners and that is special cases. When you look at 7-60 in the Manual and let's just read a couple of sentences there. It is actually a poorly worded sentence, or paragraph, it almost says the opposite of what it wants to say.

Experiences through asserting good judgment are indispensable for the successful retracement and recovery of any survey when it reaches a stage of extensive obliteration. It is an axiom among experience Cadastral surveyors that the true location of the original lines and corners can be restored if or as long as the original survey was made faithfully and was supported by a reasonably good field note record. I think you'd agree with that. That is the condition for which the basic principles have been outlined and for which the rules have been laid down.

Now what they are saying here is the rules that we have just gone through, these different methods of proportioning corners, all of those are laid down with the assumption that the original survey was made faithfully, that doesn't mean perfectly and that doesn't even mean without some fraud or cheating, but it was made faithfully and was supported by a reasonably good field note record. We at least have some record of what he said he did, how, what he measured, where he crossed this, that or the other, whatever, the usual things in a set of field notes. So that is the assumption that the rules are made on. Now the last sentence, I don't care for. It says, the rules cannot be elaborated to reconstruct a grossly erroneous survey or a survey having fictitious field notes. Now what they are saying is there really is that if it is so far out of whack that you, there is no evidence in the township, well you know, you probably going to stick with proportioning the whole think or whatever.

But what 7-60 thru 7-62 is actually trying to tell us is that there are special cases, there are situations where you might step back from the method described in the Manual to try to put the corner back in its most likely original position. Now on the slide then, let me show you this, special cases, 7-60 thru 7-62, just read that.

## Restoration of Lost Corners - Part 4

Remember what your goal is, to put it where it was or as best as you can, put it where it was. This is important as opposed to put it where it should have been. And I will show you an example of that here in a second.

Here is some likely applications where a special case would be. If the original survey is fictitious. Now if the whole township is fictitious, you've got a real problem. If there is just portions of it, I've dealt with some where it is fictitious where he just made up stuff, it is not downright fraud, you see we kind of deal with those differently at times, fraud where he was never there at all, that is different. But where there are some fictitious elements in the survey where stub outs are obvious, which is where he lied in his survey, or where there is a bust in the record, a significant drop of you know usually one or five chains depending on how they were tallying or counting the chain, the pulls of the chain.

But please notice the last line here, saving your client time and money is not a special case. I can't tell you how many thousands of those I have seen. That is not a special case. So now let's understand that in a normal township with an honest surveyor, what are you supposed to be doing up in sections one through six? Well you just run you know whatever your north or real close to north is, right, and you set your quarter corner 40 chains then you go on into the township line and you either tie into the corners that are already there or you set closing corners depending on the vintage of surveys and the situation you've got, what kind of line you are tying into. So fine, that is what they were supposed to do.

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Take a look at this slide and see a different situation. That is what the notes said for this township. This, I've redrawn it, but it is from a case up in northern Arizona here up near the town of Ashfork, right along I-40 and the surveyor said in his notes said you know I ran up this line 40 chains, put the quarter corner in, I continued up that bearing and I hit right on the corner that is up here on the township line. Now when we get up there on the ground, what do we find?

We find that he set the section corner here, he set the quarter corner here, but he never ran this line. And the reason we know that is because 20 chains away over here is where these corners are
 we're not going to, we can't change that and put it where it should that he said he hit. In the record you see he said he hit those, so have been. The fact of the matter is he didn't run these lines. He faked these lines in coming in on up there. Right? Why do you suppose he did that? Well I can tell you for sure here because there is a canyon that ran down right through there and he wasn't going to cross that canyon. So he faked his notes.

And what happened was, he had a bust in his survey way down here somewhere, you know a few miles south of here and in this township and didn't realize that he ran the whole township 20 chains west of where it should have been. So somewhere that's got to give. Now had he run these up straight, he would have realized that oh oh something is wrong. And maybe he did that but then he wasn't going to go back and redo you know two or three weeks of work in rough country out in the middle of nowhere. So he just let it go. But actually know on the ground now that these corners are sitting like this. That one is found, that one is found.

Now here is the situation, we are surveying the line between sections three and four here, this line, because, in fact I was on this case, there is a, in that canyon there is a steel, a solid steel dam and a lake behind it, and they are trying to figure out, the railroad built this back in the 1880s for water for the old cross country you know Transcontinental Railroad so they are trying to decide whether this thing is trespassing on federal land or not. And so we go out there, and some surveyor had been in there, they had hired a surveyor and he had gone and found that corner and he had

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found that one, and he couldn't find the quarter corner.
And so what did he do, well he looked in the Manual and the Manual said well single proportion on a straight line. So he single proportioned that thing on this line and ended up putting the quarter corner, you know not at midpoint, but close to it, there at that position. Now I am here to tell you that that is the last place on the planet that that quarter corner is ever going to be found is there. He either stubbed it from this way or stubbed it from that way, but there is no way that he ran that off on a 15 degree bearing and didn't know it and especially when we have this and this to determine it. So that is our special case.

Do you see what's going on here? This survey has some fictitious records. He was stubbing these corners out and you know, the fact that I didn't show all the others but the corners off that way and off the other way, they are all stubbed out too just like that except in section six it was really screwed up. So what are we going to do? Our goal is not to put it where is should have been, which is on line, our goal is to put it where it was, which is going to be over here somewhere. So I had a special case. And I thought about this for a while and I knew that that is not where I would ever say the corner was, but of course that is what you know what all the dispute was based on was whether this thing was trespassing was because of that guy's corner, well you know the Manual doesn't tell you what to do with this except 7-60 thru 7-62.

Now here is what I said it would do and after you take course number 6 on fractional sections, and I am not saying that these are fractional sections but after you take that course you'll see where I got this idea. I just said, you know what, I've got his bearing and distance here and I've got his bearing and distance here. I am going to mean those and I'm going to come up there and put that at that point. So in other words this is going to zig and zag just like the other ones it is just going to be a kind of mean of that. Because the corner is lost, the quarter corner is lost but I know that it was never set over here.

So that is what I did and so we went up here we went up the mean of those two measurements and we set, we started setting a federal monument in the ground there and as we are picking up rocks to build a mound of stones, about six feet away, we find a rock with a

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nice big one slash four on it that was face down. In other words, I was saved by the bell. I now found the original corner so I didn't have to proportion it any more but notice that my method of saying I am in a special case and I am going to come up with something that's equitable of how I am going to use those to come up with this, put me within six feet of the original corner. And actually I had looked all over the place you see I don't know how we missed that, but we had.

But now I had the original stone but you see it is because I was willing to understand and admit that I was in a special case and started thinking about equitable solution to it that had actually got me in the right neighborhood. You know and the other surveyor, I mean I understand what he did, because he was just purely following the book but he didn't pay attention to what was going on around him in that township and realize that, hey, and by the way, the USGS quadrangle showed with it with these big angle points because the USGS had gone out and found corners and they knew something was goofed up out there and when they mapped it they found enough to know that we had these big jogs.

So it wasn't like it was some big secret you know that that problem was out there. So that is one example of a special case. You know there is probably an infinite number but that is one that I wanted to share now I've got another one here that I want you to take a look at, oh I'm sorry, this is a real case of just what we were just talking about.

You see here this is a plat of a survey up in Oregon. And notice this is where the BLM found all of these like this. So you know this is the real thing and we used this plat earlier in the course to talk about setting these quarter corner if they became lost using the modified single proportion to put them there. But that is what we are talking about, is this, is that sort of thing so understand that if the original plat said these were straight, the BLM has come in and done a dependent re-survey and said, no, it's like that, and then that becomes lost well then you are now going to use the modified single proportion to put that in, not the true single proportion which would have put it midpoint on a straight line.


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So just showing you how this can happen when you are retracing a retracement. That is really what is going on there. Now, we talked about closing corners a few minutes ago and now I want to talk about a special case with a closing corner. You will recall that one of the basic principles that we talked about the first few minutes of this course was laying your chain down to their chain and I explained in closing corners how really the process that we are given, really the only process that we have violates that principle. And it is because closing corners are an odd ball situation I mean let's face it folks, the whole fundamental idea of boundary surveying is that corners don't move, right?

And it is the whole idea of the Public Land System where you come out with this grid and it is going to be there before anybody gets a patent. So the land boundaries are all fixed and nothing moves. And then they go and invent a closing corner and by definition, it moves. So you see that is why it doesn't fit very well in the system. Now here is a situation where I would back off from what 7-41 thru 7-99 in the Manual said about closing corners.

Take a look at this one, I think I changed some numbers here, yeah, now see here is our closing corner, it's lost.

We got to put it back in and remember the original surveyor came up set the quarter corner, came up set his closing corner, measured 3.83. But of course the problem is that in the $7-41$ it says well you have to go from this corner to this one, you know not only for line but also for distance. But notice what I have added here. The record says 40 chains but let's say that you get out there on the ground and you find that there is a bust in there. I find my measurement between those two standard corners is 45.22 . Now


DIAGRAM A full size version can be found in the Diagram section at the end of this study guide.


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what that means is the 22 links is just regular old chain you know was short or long right kind of stuff, you know record and measure.

But that extra 5 chains in there, they miscounted how far they went that day or that line. They miscounted. So notice I don't want, that is a hugh difference between the record and the measured. And I don't want to apply that to this 3.83 . That is going to make this 450 or something I don't know what it will do, but I mean what it's going to do is end up making that section line go off like this and that is not where it was.

So you know here is sort of situation where if I am sure about my evidence on either end and you have this much of a bust and you'd be surprised at how often you are going to come across busts in the Public Land System, especially in more remote areas. Given those factors, I think I will just go the 3.83 here, I don't think I will proportion it at all. Or maybe I have measured some other stuff down here by this surveyor enough that I could index that if I really am worried about it, but I tell you I think I just feel far more comfortable going the 3.83 than doing the proportion which expands 40 chains into 5 chains, well that is about ten percent so that would add 50 links to that, right, roughly? So you know but that is going to move it 30 feet, 40 feet, right? So I would be real cautious. So that is another example of a special case.

In this case where we got a closing corner and we want to follow the rules that they gave us but there is a bust in that senior line. So now I am going to have to come up with a different way to do this survey and let's not forget, what is our goal? It is to put it in its most likely original position, not where it should have been or where some like this, totally bogus information would put it, no, we don't want to do that.

Now I have one more special case. I mean there is probably a million possibilities. But I've got one more I want to mention to you. It is at 7-34 in the Manual. And I will just read. It's funny, it is not in 7-60 thru 7-62 course. 7-60 thru 7-62 doesn't' go into any details we're just giving you some examples. But 7-34 in the Manual is actually a place where they specifically tell you here is a special case, you know. 7-34 says another exception to the usual, wait, wait a minute, we are talking about the proportioning, single

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proportions here is what we are talking about. We are talking about some situations where you might have a special situation.

Another exception to the usual application of single proportion of measurement is occasionally important. There may be persuasive proof of a deflection in the alignment of the exterior of a township although the record shows the line to be straight. For example, measurements east and west across a range line or north and south across a latitudinal township line counting from a straight line exterior adjustment may show distances to the nearest subdivisional corners. Now what are they talking about here?

Well they go on to talk about hey you may have to modify this. Now you've already learned that because the line was run a single proportion, no a range line, it was set and a range line was established and approved by them going $40,80,40,80,40,80$, so when a corner becomes lost on that range line because the way it was done was north and south on that, you only single proportion out of it, you don't do a double proportion or some other thing, you do single proportion because that is the way the corner was established. And that is one of the basic rules of proportioning by the way is how was the corner established.

You know what measurements and what chain created that and that is usually independent, not always, but usually independent of other chains that may close into it later. But here they are showing us an exception. Now let's take a look at the slide.

Let's say that you are, this is a range line just like they said and that the record says it is straight all the way for six miles, okay, but let's say you discover, you go out there and you found this corner but you can't find this corner and you found this corner. If you do a single proportion on this, which is what the book said to do, then you are going to put a corner over here somewhere.

That is where you will end up setting it, just roughly over there. But what they are talking about here is, hey, here is an exception to the single proportion on a range line. Perhaps you are going to look at the fact that when you come from this quarter corner back, it puts it way over here and when you come from this quarter corner, which by the way is a separate survey here from here. Usually. And it also puts you over here now you've got some

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pretty conclusive proof that he did not in fact run a straight line through there but in fact had abandoned it. That is what this slide is showing you is a special case. So the rule is you single proportion on the straight line on a range line or a township line, well I say a straight line on a township, the north and south boundary of a township would be on the longitudinal curve but you know it's a single proportion.

But here is a place where the Manual 7-34 says you know what you might want to double proportion this instead of single proportioning it using these distances, all four of them you see, because your side distances agree that that corner must have been way over there. Now understand, if I was in here doing this and 40 chains from here put me here but 40 chains from here put me over here. I wouldn't do this. What we are talking about is really this survey and this survey which are separate from each other and separate from this survey, if they agree at some place that looks like you know within a few feet looks like boy its way off from where we are looking or where the single proportion is that is what $7-34$ is talking about.

And that is just another example of a special case that we would want to look at. So you know there can be an infinite number of special cases I suppose or circumstances. I haven't found too many, I have probably proportioned in corners, lost corners, in my whole career may I don't know five, six, seven times where I did it differently than the Manual says to do it because I had a special case.

So special cases don't come along always and as we said at the beginning of this course, you really can't understand the exceptions to the rules until you understand the rules like the back of your hand. Right? So we've given you all the rules and we then we've have given you here three examples of some special cases just so that you can kind of see, we still try to stay within, we still try to use one of the other rules, we still try to in any way we can to make it something in the book in a sense of I just you know didn't pull some new solution out of somewhere and say hey here is my new theory on how to proportion corners when something is screwed up.

No, we are careful here. And try to make it make sense and you

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know that is something that I can justify. Now I will mention you know if you are ever really dealing with a special case, you need to write that up, you need to document that and I believe on your survey document especially if it ends up being an actual plat or notes, or whatever you need to have an explanation as to why you differed from the Manual method. Because every surveyor after you that comes along and sees that for the next thousand years is going to wonder why and if you give them a good explanation, they may not even agree with you, but they will understand what you did.

They won't have to guess, they don't have to think that you just blew it, or you didn't know what you were doing. They'll realize that you addressed the issue and you tried to figure it out. Well Ron, between the two of us we have just about beat them over the head with restoration of lost corners. I think we have. And we have tried to show you all of the methods that are in the Manual and the correct way to do those and we've shown you special cases and we have also given you some guidance I hope on some other things with your curvature discussion, and the Grant Boundary which I found that real interesting because I have never actually seen that discussed. So we've given you a lot of different things here.

Well and you know I think that this is one of the very most important parts of this training, learning to properly reestablish corners, the Manual can be difficult to comprehend and understand sometimes in those sections about the restoration methods and so hopefully we have presented them in a method that is a little more understandable not only in how to do it but when, when to do it and why. So you know now that we have shown you all the proper math and those things, you will recall that in the very beginning of this course we told you we hope you hate proportioning.

And Ken Witts quote you know, "Proportioning is an admission that our profession has failed somewhere." And really we want to keep that in mind because we have shown you exactly how to do it by the book, by the law, and some guidance on special cases, whatever. But let's talk about for a moment now that you have done your proportion, you've used the right math, the right basis of bearings, the right control, all of those things, now you are

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ready to set that corner point.

Let's just review on the slide what you really should do after the proportion. That is, always look again. Remember that no matter how perfectly you computing the position and no matter how perfect your measurements or your control is, evidence is always going to control the corner point, not the numbers. I can't tell you how many times I've done a proportion and then looked again and found evidence that I had missed before. And so you know as we saw in a couple of examples I showed you earlier in the course, sometimes just doing the proportion or really thinking about what I've got led me to the evidence because it got me into the right neighborhood and got me to paying attention to things that I should and got me you know a few feet different from where I had been looking and thinking where this corner was really going to be.

## After the Proportion

- ALWAYS LOOK AGAIN!

Evidence will always control the corner point; not numbers

- Set a durable monument, with proper marks, as required
- Establish new accessories per the Manual
- Make a public record showing all details, as required

So evidence is always going to control that. You are going to have to set durable monument depending upon what kind of surveys you are doing, if it is Federal survey, we've got our requirements. If it is just something you are doing, I encourage you to still set a durable monument.

Put the proper marks on it even if its just you out there, not on Indian country or Public Domain but you're setting a center south $16^{\text {th }}$, I encourage you to mark it just like chapter 4 of the Manual says. That is the way that it has been done for a hundred years and really that is the real true sign of a real professional someone who really knows what they're doing in the Public Land System and then establish some new accessories. Do it like the Manual says at a minimum, one per section, or at least two on some of these interior corners or other things.

That may be bearing trees or just swing ties or one thing or another to more permanent objects or reference monuments that you set but the whole idea of this is that if we had to proportion because the profession failed, let's not fail the profession again, let's not set it up where we did all this computation and did this great job and used the right control and everything and then didn't set enough evidence that it will be found ten years or fifty years from now. So let's not fail it again, so you want to establish those

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accessories and then whether your state law says it or not I strongly encourage you to make a public record of everything you did and of course if you are doing CFedS and that sort of thing, we've got standards and guidelines on that but just any kind of survey.

Make a public record showing where everything is, telling us how you did it, why you did it. You know, I'll say it again, when we see your record, when I say we you know me and any other surveyor, when I see your record, I may not agree with what you did, but its really best if there is no question in my mind what you did, in other words, now help me understand, you know what I think that for you and your client's sake whoever it is, you greatly increase the possibilities that future surveyors will accept your position. The more information you give that we don't have to guess what you did or think that you don't know how to do this right, you have actually given us the information that tells us how you did it and explains it and we may say well that's probably not how I would have done it, but Ron did that ten years ago, and I'm going to leave it, you know it looks good to me.

So after the proportion, here's the point, yeah you did the math, go look again, then either way whether you find something or not, set a durable monument, mark it, take accessories, make a public record. Well that is kind of our prepared comments here, Ron do you have any closing comments. Well, no, I think that we have got this covered and hopefully it is been a good session for you. I think Dennis did a great job on this and hopefully this will be helpful for you in your career in the future as you progress. So I've got one more piece of advice for you, no matter what you do, no matter what method of proportioning or whatever you do, there is one thing that I would rather you did and that is just find the corner, okay just find the corner.

That is better to any kind of a proportion, to any kind of an adjustment, any kind of control network, anything it is best to just find the evidence and as you heard in the course previous to this one, that is the heart and soul of Cadastral Survey and so I hope you got something out of this and your next module, your next course will be on water boundaries and we'll run into you later in the program.

## Restoration of Lost Corners - Part 4



PRoblem Before moving on to the next topic, complete the "Calculate Proportionate Positions of Corners \& Adjust Meander Line" which you can access from the course description page.


It's time to take the Course 4 Quiz.
You can access the quiz from the CFedS website.


## DIAGRAM






## EXERCISE

## Grant Boundary and Compass Rule

|  | Record | Retrace |
| :---: | :---: | :---: |
| Reservation |  | Found Orig. <br> N. 14041.18 <br> E. 11233.54 D <br> Reservation |
|  | 2.67 chs. <br> C <br> Calculate the position of the lost corners using the grant boundary method and the compass rule. |  |

## Grant Boundary Calculation

Inverse between controlling corners: $\quad$ Record $=\mathrm{N} .3865 .42 \mathrm{ft} .$, E. 1325.41 ft. N. $18^{\circ} 55^{\prime} 35 "$ E., 4086.33 ft .

Retrace $=$ N. $4041.18 \mathrm{ft} .$, E. 1233.54 ft .
N. $16^{\circ} 58^{\prime} 28^{\prime \prime}$ E., 4225.25 ft .

Angular rotation: $1^{\circ} 57^{\prime} 07^{\prime \prime}$ counter-clockwise ( from record to retrace)
Line A-B: N. $15^{\circ} 34^{\prime}$ E. - $1^{\circ} 57^{\prime} 07^{\prime \prime}=\mathbf{N . ~ 1 3}{ }^{\circ} \mathbf{3 6}{ }^{\prime} \mathbf{5 0} "$ E.
Line B-C: N. $90^{\circ} 00^{\prime}$ E. - $1^{\circ} 57^{\prime} 07^{\prime \prime}=\mathbf{N . ~ 8 8}{ }^{\circ} \mathbf{0 2}$ ' 53" E.
Line C-D: N. $0^{\circ} 00^{\prime} \mathrm{W} .-1^{\circ} 57^{\prime} 07^{\prime \prime}=\mathbf{N .} 1^{\circ} 57^{\prime} 07^{\prime \prime} \mathbf{W}$.
Proportion: 4225.25 ft .(retrace) $\div 4086.33 \mathrm{ft}$.(record) $=1.033996$
$1.033996 \times 1822.92 \mathrm{ft}$. ( 27.62 chs.$)=\mathbf{1 8 8 4 . 8 9} \mathbf{f t}$.
" $\quad \mathrm{x} 836.22 \mathrm{ft}$. $(12.67 \mathrm{chs})=.\mathbf{8 6 4 . 6 5} \mathbf{f t}$.
" $\quad$ x 2109.36 ft . ( 31.96 chs. ) $=\mathbf{2 1 8 1 . 0 7} \mathbf{f t}$.
Lat. and Dep. of each course:
Line A-B: N. $13^{\circ} 36^{\prime} 50^{\prime \prime}$ E., 1884.89 ft N. 1831.93 ft ., E. 443.66 ft .
Line B-C: N. $88^{\circ} 02^{\prime} 53^{\prime \prime}$ E., 864.65 ft . N. 29.45 ft . E. 864.15 ft .
Line C-D: N. $1^{\circ} 57^{\prime} 07^{\prime \prime}$ W., 2181.07 ft . N. 2179.80 ft . W. 74.29 ft .

## Proportionate Position Corners B and C

Corner B: N. $10000.00+1831.93 \mathrm{ft} .=$ N. $11831.93 \quad$ E. $10000.00+443.66 \mathrm{ft} .=$ E. 10443.66
Corner C: N. $11831.93+29.45 \mathrm{ft} .=\mathbf{N} .11861 .38 \quad$ E. $10443.66+864.15 \mathrm{ft} .=\mathbf{E . 1 1 3 0 7 . 8 1}$

## Compass Rule Calculation

Misclosure Record vs. Retrace
Record coordinates at D: Lat. N. 3865.41 ft ., Dep. E. 1325.42 ft .
Retrace coordinates at D: Lat. N. 4041.18 ft . Dep. E. 1233.54 ft .
N. 175.77 E.-91.88

Length of all the courses: 4768.50 ft .
Correction to course AB: 1822.92 ft . ( 27.62 chs.) $\div 4768.50=0.382284$
$0.382284 \times \mathrm{N} .175 .77=\mathrm{N} .67 .19$
0.382284 x E. $-91.88=$ E. -35.12

Correction to course BC: 836.22 ft . ( 12.67 chs.) $\div 4768.50=0.175363$
0.175363 x N. $175.77=\mathrm{N} .30 .83$
0.175363 x E. -91.88 = E. -16.11

Correction to course CD: 2109.36 ft . ( 31.96 chs.) $\div 4768.50=0.442353$
0.442353 x N. $175.77=\mathrm{N} .77 .75$
$0.442353 \times$ E. $-91.88=$ E. -40.64

## Record Lat and Dep. of each course plus correction

Course A-B: N. $1756.05+67.19=\mathbf{N} .1823 .24 \quad$ E. $489.20-35.12=\mathbf{E . 4 5 4 . 0 8}$
N. $13^{\circ}$ 59' 06" E., 1878.93 ft .

Course B-C: N. $0.00+30.83=\mathbf{N} .30 .83 \quad$ E.836.22 - $16.11=\mathbf{E . 8 2 0 . 1 1}$
N. $87^{\circ} \mathbf{5 0}{ }^{\prime}$ 50" E., 820.69 ft.

Course C-D: N. $2109.36+77.75=\mathbf{N} .2187 .11 \quad$ E. $0.00-40.64=$ E.-40.64
N. $1^{\circ} 03$ ' 52" W., 2187.49 ft.

## Proportionate Position Corners B and C

Corner B: N. $10000.00+1823.24 \mathrm{ft} .=$ N. $11823.24 \quad$ E. $10000.00+454.08 \mathrm{ft} .=$ E. 10454.08
Corner C: N. $11823.24+30.83 \mathrm{ft} .=\mathbf{N} .11854 .07$
E. $10454.08+820.11 \mathrm{ft} .=\mathbf{E} .11274 .19$



[^0]:    "2-1. The law prescribes the chain as the unit of linear measure for the survey of the public lands. All returns of measurements in the rectangular system are made in the true horizontal distance in miles, chains and links...."
    "2-17. The direction of each line of the public land surveys is determined with reference to the true meridian as defined by the axis of the earth's rotation. Bearings are stated in terms of angular measure referred to the

